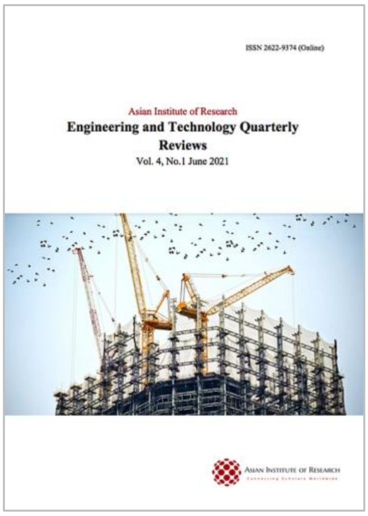

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Application of Uniformly Valid Shell Theory

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Abstract
For the purpose of demonstrating the applicability of the previously derived theories, the problem of a laminated circular cylindrical shell under internal pressure and edge loadings will be examined. The cylinder is assumed to consist of boron/epoxy composite layers. Each layer is taken to be homogeneous but anisotropic with an arbitrary orientation of the elastic axes. We need not consider the restriction of the symmetry of the layering due to the non-homogeneity considered in the original development of the theory expressed by the constitutive equations. Thus, each layer can possess a different thickness.

Keywords: Shell Theory, circular cylindrical shell, applicability

INTRODUCTION

We assume here that the contact between layers is such that the strains are continuous functions of the thickness coordinate. As the C's are piecewise continuous functions, the in-plane stresses are also. We would expect them to be discontinuous at the juncture of layers of dissimilar materials. The transverse stresses are continuous functions of the thickness coordinate. Although as mentioned above the theory developed can consider, random layering, numerical results are to be carried out for a four layer symmetric angle ply configuration (see Appendix C for asymmetric cross-ply layering). For this configuration the angle of elastic axes y is oriented at $+\gamma$, $-\gamma$, $-\gamma$, $+\gamma$, with the shell axis and the layers are of equal thickness. The coefficients Aij Bij Dij Eij and Fij for this configuration can be determined by making use of the results for layered shells developed in Appendix B. Let the cylinder be subjected to an internal pressure p, an axial force per unit circumferential length N and a torque T. The axial force is taken to be applied at r=a+H such that a moment N(H-d) is produced about the reference surface r=a+d. We introduce dimensionless external force and moments as follows:

$$\overline{N} = N/(\sigma\lambda a)$$

$$\overline{M} = [N(H-d)]/(\sigma\lambda^2 a)$$

$$\overline{T} = T/[2\pi\sigma\lambda^2 a^3(1+d/a)]$$

Equation (1)

The cylinder is taken to be clamped at both ends but free to rotate and extend axially at one end. The edge conditions can thus be written as

$$v_r = V_{r,x} = V_z = V_\theta = 0 \quad (x=0,\ y=d/h)$$

$$v_r = 0,\ \overline{N}_z = \overline{N},\ \overline{M}_z = \overline{M}, \quad (x=l,\ y=d/h)$$

$$(1+d/a)\overline{N}_{z\theta} + \lambda\overline{M}_{z\theta} = \overline{T}$$

Equation (2)

Here, **l** is the dimensionless length of the cylinder.

The above edge conditions are assumed to represent a close approximation to the test set-up used for obtaining the mechanical properties of the composites. In the theories developed in the previous chapters, the distance d at which the stress resultants were defined was left arbitrary. We now choose it to be such that there exists no coupling between N_z and K_1 and M_z and ε_{1d}

This can be achieved by setting the first component of sub- matrix [B] equal to zero,

$$B_{ij} = 0$$

This yield

$$d/h = \underline{B}_{11} / \underline{A}_{11}$$

Equation (3)

For a homogeneous, isotropic material we obtain, for example,

$$d/h = 1/2$$

As the loading applied at the end of the shell is axi-symmetric, all the stresses and strains are also taken to be axi-symmetric. We thus can set all the φ derivatives in the expressions for the stresses and strains and in the equations for the displacements equal to zero. The resulting simplifications and the general solution corresponding to each of the derived theories is now given. As the theory corresponding to axial length scale $a(a/h)^{1/2}$ and circumferential length scale a has been used in the analysis of long cylindrical shells with edges of the form φ = constant, it will not be considered.

a) <u>Theory associated with Length Scales a</u>

<u>Strain-Displacement Relations</u>

$$\varepsilon_1 = V_{z,x}$$

$$\varepsilon_2 = V_r$$

$$\varepsilon_{12} = V_{\theta,x}$$

$$\varepsilon_1 = V_{z,x}$$

$$\varepsilon_2 = V_r$$

$$\varepsilon_{12} = V_{\theta,x}$$

Governing Equations

$$\underline{A}_{11}V_{z,xx} + \underline{A}_{12}V_{r,x} + \underline{A}_{13}V_{\theta,xx} = 0$$

$$\underline{A}_{13}V_{z,xx} + \underline{A}_{23}V_{r,x} + \underline{A}_{33}V_{\theta,xx} = 0$$

$$\underline{A}_{12}V_{z,xx} + \underline{A}_{22}V_{r} + \underline{A}_{23}V_{\theta,x} = p^*$$

Equation (4)

By combining the first two equations, we obtain expressions relating Vz and Vr and V_θ and Vr , respectively,

$$V_{z,xx} = \mathbf{B}\, V_{r,x}$$

$$V_{\theta,xx} = \mathbf{C}\, V_{r,x}$$

Equation (5)

where the coefficients R and £ are defined as follows:

$$\mathbf{B} = [\underline{A}_{13}\underline{A}_{23} - \underline{A}_{12}\underline{A}_{33}]/[\underline{A}_{11}\underline{A}_{33} - \underline{A}_{13}^2]$$

$$\mathbf{C} = [\underline{A}_{11}\underline{A}_{23} - \underline{A}_{13}\underline{A}_{12}]/[\underline{A}_{13}^2 - \underline{A}_{11}\underline{A}_{33}]$$

$$V_{z,x} = \mathbf{B}\, V_r + C_1^*$$

$$V_{\theta,x} = \mathbf{C}\, V_r + C_2^*$$

Equation (6)

Integrating equations with respect to x yields where C_1* and C_2* are constants of integrations.

On substituting the equations into the last equation of (4) we obtain,

$$N_1 V_r = p^* - \underline{A}_{21}C_1^* - \underline{A}_{23}C_2^*$$

Where

$$N_1 = \underline{A}_{12}\mathbf{B} + \underline{A}_{22} + \underline{A}_{23}\mathbf{C}$$

Equation (10) shows that the radial deformation depends on the pressure P* and the material properties of the individual layers. These effective properties in turn depend on the angle y between the elastic axis of the material of each layer and the shell axis.

a : Inside Radius of Cylindrical Shell
h : Total Thickness of the Shell Wall
d; Distance (thickness) from Inside Radius and to mechanical neutral surface
Si : Radius of Each Layer of Wall (I = 1, 2, 3 -- - to the number of layer)
L : Longitudinal Length Scale to be defined, Also Actual Length of the Cylindrical Shell
Π : Circumferential Length Scale of cylindrical shell to be defined
Ei : Young’s Moduli in I Direction
Gij : Shear Moduli in i-j Face
Sij : Compliance Matrix of Materials of Each Layer
r : Radial Coordinate
Π : Circumferential Length Scale to be defined
Ɣ : Angle of Fiber Orientation
σ : Normal Stresses
ε : Normal Strains
z, θ, r : Generalized Coordinates in Longitudinal, Circumferential and Radial

Directions Respectively
τ : Shear Stresses
εij : Shear Strains in i-j Face
λ ; Shell Thickness / Inside Radius (h/a)
Cij : Elastic Moduli in General
X, φ, Y : Non Dimensional Coordinate System in Longitudinal, Circumferential and Radial Directions Respectively

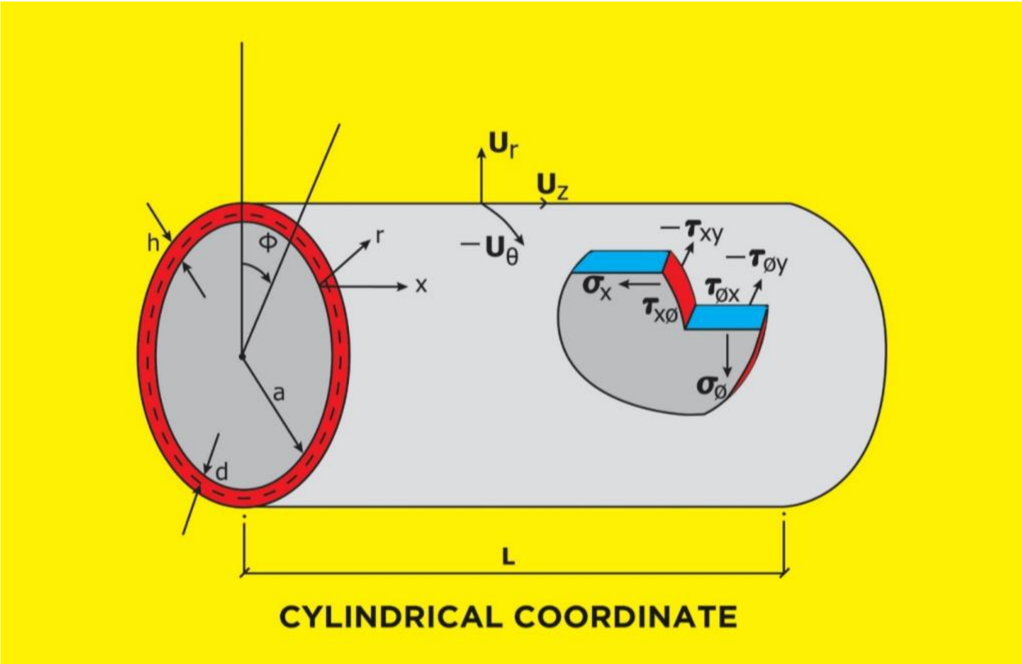

Figure 1: A cylindrical shell showing dimensions, deformations and stresses

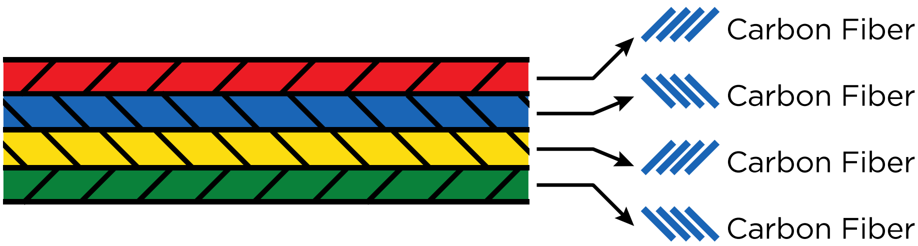

Figure 2: Anisotropic materials

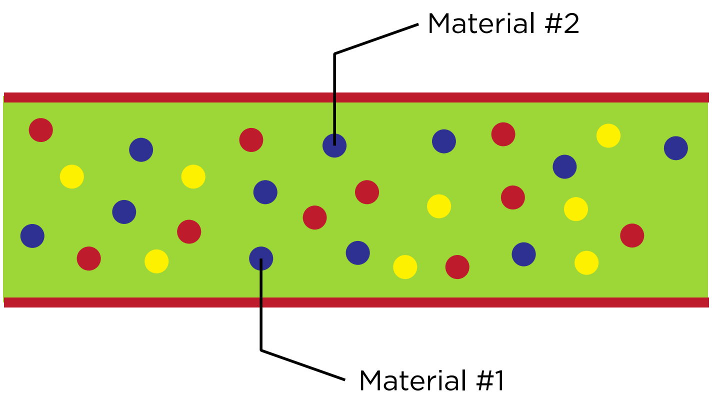

Figure 3: Non-homogenous materials

Note that since we have only four unknown constants, Ci*. (i = 1, 4), the solution of this theory can accommodate four boundary conditions. We therefore must abandon some of the boundary conditions specified in equations (9.3). The chosen two boundary conditions are:

$$v_z = v_\theta = 0 \qquad (x = 0)$$

$$\overline{N}_z = \overline{N}, \ (1+d/a)(\overline{N}_{z\theta}+\lambda\overline{M}_{z\theta})=\overline{T} \quad (x = \ell)$$

Equation (7)

where the external forces, >N" and T are as defined by equation (2) The equation (10) thus has a particular solution only

$$v_r^p = (p^* - \underline{A}_{21}C_1^* - \underline{A}_{23}C_2^*)/N_1$$

Equation (8)

From now on the superscript p_wm indicate the particular solution.

b) Theory Associated with Axial Length Scale $(ah)^{1/2}$ and Circumferential Length Scale a

Strain-Displacement Relations

$$\varepsilon_1 = V_z$$

$$\varepsilon_2 = V_r$$

$$\varepsilon_{12} = V_{\theta,x}$$

$$K_1 = -V_{r,xx}$$

$$K_2 = 0$$

$$K_{12} = 0$$

C*_ and C| arise from the integration

Equation (9)

Governing Equations

$$\underline{A}_{11}V_{z,xx} + \underline{A}_{12}V_{r,x} - \underline{B}_{11}V_{r,xxx} = 0$$

$$\underline{A}_{33}V_{\theta,xx} = 0$$

$$\underline{A}_{12}V_{z,x} - \underline{B}_{12}V_{r,xx} + \underline{A}_{22}V_r + \underline{D}_{11}V_{z,xxx} + \underline{D}_{12}V_{r,xx} - \underline{E}_{11}V_{r,xxxx} = p^*$$

Equation (10)

From the first two equations of above we obtain the following relations for V and V_Q:

$$V_{z,x} = \mathbf{D}\,V_{r,xx} + C_3^* - \mathbf{E}\,V_r$$

$$V_\theta = C_1^* x + C_2^*$$

Equation (11)

where $C^*_{i's}$ (i = l,3) are the integration constants to be determined later and the coefficients **D** and **E** are defined as follows:

$$\mathbf{D} = \underline{B}_{11}/\underline{A}_{11}$$

$$\mathbf{E} = \underline{A}_{12}/\underline{A}_{11}$$

We can now express the third equation of (15) in terms of a single variable V by substituting (16) into it. This yield

$$N_1 V_{r,xxxx} - 2N_2 V_{r,xx} + N_3 V_r = p^* - \underline{A}_{21} C^*_3$$

Equation (12)

where the constants N_i (i = 1.2.3) are as follows:

$$N_1 = \underline{D}_{11}\,\mathbf{D} - \underline{E}_{11}$$

$$N_2 = (\underline{B}_{12} + \underline{D}_{11}\,\mathbf{E} - \underline{A}_{21}\,\mathbf{D} - \underline{D}_{12})\ /2$$

$$N_3 = \underline{A}_{22} - \underline{A}_{21}\,\mathbf{E}$$

The homogeneous solution of equation (9.18) can be expressed as,

$$V_r^H = \exp(-N_5 x)(A_1 \cos N_6 x + A_2 \sin N_6 x)$$
$$+ \exp(-N_5 \xi)(A_3 \cos N_6 \xi + A_4 \sin N_6 \xi)$$

Equation (13)

Here, the A_i's are constants to be determined by the edge conditions specified earlier, N^ and N, are given by

Governing Equations

$$\underline{A}_{11} V_{z,xx} + \underline{A}_{12} V_{r,x} - \underline{B}_{11} V_{r,xxx} = 0$$

$$\underline{A}_{33} V_{\theta,xx} = 0$$

$$\underline{A}_{12} V_{z,x} - \underline{B}_{12} V_{r,xx} + \underline{A}_{22} V_r + \underline{D}_{11} V_{z,xxx} + \underline{D}_{12} V_{r,xx} - \underline{E}_{11} V_{r,xxxx} = p^*$$

Equation (14)

From the first two equations above, we obtain the following relations for V and V_Q:

$$V_{z,x} = \mathbf{D}\, V_{r,xx} + C^*_3 - \mathbf{E}\, V_r$$

$$V_\theta = C^*_1 x + C^*_2$$

where $C^*_{i's}$ (i = l,3) are the integration constants to be determined later and the coefficients **D** and **E** are defined as follows:

$$\mathbf{D} = \underline{B}_{11}/\underline{A}_{11}$$

$$\mathbf{E} = \underline{A}_{12}/\underline{A}_{11}$$

We can now express the third equation of (9.15) in terms of a single variable V by substituting (9.16) into it. This yield,

$$N_1V_{r,xxxx} - 2N_2V_{r,xx} + N_3V_r = p^* - \underline{A}_{21}C_3^*$$

Equation (15)

where the constants N_i (i = 1.2.3) are as follows:

$$N_1 = \underline{D}_{11}\mathbf{D} - \underline{E}_{11}$$

$$N_2 = (\underline{B}_{12} + \underline{D}_{11}\mathbf{E} - \underline{A}_{21}\mathbf{D} - \underline{D}_{12})/2$$

$$N_3 = \underline{A}_{22} - \underline{A}_{21}\mathbf{E}$$

Equation (16)

The homogeneous solution of above equation can be expressed as,

$$V_r^H = \exp(-N_5x)(A_1\cos N_6x + A_2\sin N_6x) + \exp(-N_5\xi)(A_3\cos N_6\xi + A_4\sin N_6\xi)$$

Equation (17)

Here, the A_i's are constants to be determined by the edge conditions specified. N^ and N, are given by

$$N_5 = (N_3/N_1)^{1/4}\cos(\alpha/2)$$

$$N_6 = (N_3/N_1)^{1/4}\sin(\alpha/2)$$

Equation (18)

where

$$\alpha = \tan^{-1}[(N_1N_3 - N_2^2)/N_2^2]^{1/2}$$

and £, is the dimensionless coordinate originating from the far edge of the shell defined as,

$$\xi = L/(ah)^{1/2} - x$$

The particular solution of

$$V_r^p = (p^* - \underline{A}_{21}C_3^*)/N_3$$

Thus the complete solution is given by

$$V_r = V_r^p + V_r^H$$

c) Theory Associated with Length Scales $(ah)^{1/2}$

Strain-Displacement Relations

$$\varepsilon_1 = V_{z,x}$$

$$\varepsilon_2 = V_r$$

$$\varepsilon_{12} = V_{\theta,x}$$

$$K_1 = -V_{r,xx}$$

$$K_2 = 0$$

$$K_{12} = 0$$

Equation (19)

Governing Equations

$$\underline{B}_{11} V_{r,xxx} - \underline{A}_{12} V_{r,x} - \underline{A}_{11} V_{z,xx} - \underline{A}_{13} V_{\theta,xx} = 0$$

$$\underline{B}_{13} V_{r,xxx} - \underline{A}_{23} V_{r,x} - \underline{A}_{13} V_{z,xx} - \underline{A}_{33} V_{\theta,xx} = 0$$

$$\underline{E}_{11} V_{r,xxxx} - (\underline{D}_{12} - \underline{B}_{12}) V_{r,xx} - \underline{A}_{22} V_r - \underline{D}_{13} V_{\theta,xxx} - \underline{A}_{23} V_{\theta,x} - \underline{D}_{11} V_{z,xxx} - \underline{A}_{12} V_{z,x} = -p^*$$

The first two equations of the above will be as follows when solved for V_z and V_θ yield,

$$V_{z,x} = \mathbf{P}\, V_{r,xx} + C_3^* - \mathbf{Q}\, V_r$$

$$V_{\theta,x} = \mathbf{F}\, V_{r,xx} + C_1^* - \mathbf{G}\, V_r$$

Equation (20)

where the coefficients F, G, P. Q are defined as follows:

$$\mathbf{G} = (\underline{A}_{13}\underline{B}_{11} - \underline{A}_{11}\underline{B}_{13})/\mathbf{\Omega}$$

$$\mathbf{F} = (\underline{A}_{12}\underline{A}_{13} - \underline{A}_{11}\underline{A}_{33})/\mathbf{\Omega}$$

$$\mathbf{Q} = (\underline{A}_{13}\underline{B}_{13} - \underline{A}_{33}\underline{B}_{11})/\mathbf{\Omega}$$

$$\mathbf{P} = (\underline{A}_{13}\underline{A}_{23} - \underline{A}_{12}\underline{A}_{33})/\mathbf{\Omega}$$

and Q is given by

On substituting the above equations into the third equation of constitutive equations we obtain differential equation for V only,

$$N_1 V_{r,xxxx} - 2N_2 V_{r,xx} + N_3 V_r = N_4(-p^* + \underline{A}_{23} C_1^* + \underline{A}_{12} C_2^*)$$

where C_1^* and C_2^* are constants of integration and

$$N_1=(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}^2)\underline{E}_{11}-(\underline{A}_{11}\underline{B}_{13}-\underline{A}_{13}\underline{B}_{11})\underline{D}_{13}-(\underline{A}_{33}\underline{B}_{11}-\underline{A}_{13}\underline{B}_{13})\underline{D}_{11}$$

$$N_2=-\frac{1}{2}\,[(\underline{B}_{12}-\underline{D}_{12})(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}^2)+(\underline{A}_{11}\underline{A}_{23}-\underline{A}_{12}\underline{A}_{13})\underline{D}_{13}+(\underline{A}_{12}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{23})\underline{D}_{11}$$

$$-(\underline{A}_{11}\underline{B}_{13}-\underline{A}_{13}\underline{B}_{11})\underline{A}_{23}-(\underline{A}_{33}\underline{B}_{11}-\underline{A}_{13}\underline{B}_{13})\underline{A}_{12}]$$

$$N_3=(\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}^2)(\underline{A}_{23}-\underline{A}_{22})+(\underline{A}_{11}\underline{A}_{22}-\underline{A}_{12}\underline{A}_{13})\underline{A}_{23}+(\underline{A}_{12}\underline{A}_{33}-\underline{A}_{13}\underline{A}_{23})\underline{A}_{12}$$

$$N_4=\underline{A}_{11}\underline{A}_{33}-\underline{A}_{13}^2$$

Equation (21)

The homogeneous solution of equation is representable in a similar fashion,

$$V_r^H = \exp(-N_5x)(A_1\cos N_6x + A_2\sin N_6x)$$

$$+\exp(-N_5\xi)(A_3\cos N_6\xi + A_4\sin N_6\xi)$$

where A_1's, and N_5 and N_6 are as defined previously, The particular solution is given by

$$V_r^P=(-p^*+\underline{A}_{23}C_1^*+\underline{A}_{12}C_2^*)(N_4/N_3)$$

The complete solution is then

$$V_r = V_r^P + V_r^H$$

Equation (22)

d) Uniformly Valid Theory

Strain-Displacement Relations

$$\varepsilon_1 = U_{z,z}$$

$$\varepsilon_2 = U_r/a$$

$$\varepsilon_{12}=U_{\theta,z}$$

$$K_1 = -U_{r,zz}$$

$$K_2 = 0$$

$$K_{12}= 2U_{\theta,z}/a$$

Governing Equations

$$\underline{A}_{11} aU_{z,zz} + \underline{A}_{13} aU_{\theta,zz} + \underline{A}_{12} U_{r,z} - a\lambda[\underline{B}_{11} aU_{r,zzz} + (\underline{B}_{12} + 2\underline{B}_{33})(1/a)U_{r,z} - 2\underline{B}_{13} U_{\theta,zz}] = 0$$

$$\underline{A}_{13} aU_{z,zz} + \underline{A}_{33} aU_{\theta,zz} + \underline{A}_{23} U_{r,z} - a\lambda[\underline{B}_{13} aU_{r,zzz} + 2\underline{B}_{23}(1/a)U_{r,z}] = 0$$

$$\underline{A}_{12} U_{z,z} + \underline{A}_{22}(1/a)U_{r} + \underline{A}_{23} U_{\theta,z} - a\lambda[\underline{B}_{12} U_{r,zz} + \underline{B}_{22}(1/a^2)U_{r} - 2\underline{B}_{13}(1/a)U_{\theta,z}]$$
$$- a^3\lambda^2 \underline{E}_{11} U_{r,zzzz} + a\lambda(a\underline{D}_{11} U_{z,zzz} + a\underline{D}_{13} U_{\theta,zzz} + \underline{D}_{12} U_{r,zz}) = p$$

Equation (23)

The first two equations of above give us the following relations for U_z and U_θ:

$$U_{\theta,z} = \mathbf{R} a\lambda U_{r,zz} - \mathbf{S}\frac{1}{a}U_r + C_1^*$$

$$U_{z,z} = \mathbf{T} a\lambda U_{r,zz} - \mathbf{W}\frac{1}{a}U_r + C_2^*$$

where C_1^* and C_2^* are the constants of integrations and the coefficients R, S, T, W are defined as follows:

$$\mathbf{R} = (\underline{B}_{13}\underline{A}_{11} - \underline{B}_{11}\underline{A}_{13})/\boldsymbol{\psi}$$

$$\mathbf{S} = [\lambda\underline{A}_{13}(\underline{B}_{12} + 2\underline{B}_{33}) + \underline{A}_{11}\underline{A}_{23} - \underline{A}_{13}\underline{A}_{12} - 2\lambda\underline{B}_{23}\underline{A}_{11}]/\boldsymbol{\psi}$$

$$\mathbf{T} = (\underline{B}_{11}\underline{A}_{33} - \underline{A}_{13}\underline{B}_{13} - 2\lambda\underline{B}_{13}^2)/\boldsymbol{\psi}$$

$$\mathbf{W} = [\underline{A}_{33}\underline{A}_{12} - \lambda\underline{A}_{33}(\underline{B}_{12} + 2\underline{B}_{33}) - \underline{A}_{13}\underline{A}_{23} - 2\lambda\underline{B}_{13}\underline{A}_{23} + 2\lambda\underline{B}_{23}\underline{A}_{13} + 4\lambda^2\underline{B}_{13}\underline{B}_{23}]/\boldsymbol{\psi}$$

and

$$\boldsymbol{\psi} = \underline{A}_{11}\underline{A}_{33} - \underline{A}_{13}^2 - 2\lambda\underline{B}_{13}\underline{A}_{13}$$

On substituting (9.38) into the third equation of (9.36) we obtain a differential equation for U only,

$$N_1 U_{r,zzzz} - 2N_2 U_{r,zz} + N_3 U_r = p - N_4$$

Equation (24)

where

$$N_1 = a^3\lambda^2(\underline{D}_{13}\mathbf{R} + \underline{D}_{11}\mathbf{T} - \underline{E}_{11})$$

$$N_2 = a\lambda[\underline{B}_{12} - \underline{D}_{12} - \mathbf{T}\underline{A}_{12} - \mathbf{R}(\underline{A}_{23} + 2\lambda\underline{B}_{13}) + \underline{D}_{13}\mathbf{S} + a^2\lambda\underline{D}_{11}\mathbf{W}]/2$$

$$N_3 = [-\mathbf{W}\underline{A}_{12} - \mathbf{S}(\underline{A}_{23} + 2\lambda\underline{B}_{13}) + \underline{A}_{22} - \lambda\underline{B}_{22}]/a$$

$$N_4 = \underline{A}_{12}C_2^* + (\underline{A}_{23} + 2\lambda\underline{B}_{13})C_1^*$$

APPPLICATION

The homogeneous solution can be written as

$$U_r^H = \exp(-N_5 z)(A_1 \cos N_6 z + A_2 \sin N_6 z) + \exp(-N_5 \eta)(A_3 \cos N_6 \eta + A_4 \sin N_6 \eta)$$

where A_i (i = l, 2, 3, 4) are the constants of integration to be determined and N_5, N_6 are as defined previously. As we are now dealing with actual coordinates (except for the thickness coordinate), we define n as follows:

$$\eta = L - z$$

where L is the actual length of the cylinder. The constants of integration are to be determined from the following edge conditions:

$$U_r = U_{r,z} = U_z = U_\theta = 0 \qquad (z = 0,\ y = d/h)$$

$$U_r = 0,\ N_z = N,\ M_z = N(H-d),\ T = 2\pi a(1+d/a)[M_{z\theta} + a(1+d/a)N_{z\theta}]$$

$$(z = L,\ y = d/h)$$

The particular solution is

$$U_r^p = (p - N_4)/N_3$$

and the complete solution of equation is then given by

$$U_r = U_r^p + U_r^H$$

Equation (25)

Having obtained the above solutions for each of the theories, numerical calculations are now carried out for a shell of the following dimensions:

We thus have a thickness to radius ratio of

$$\lambda = 0.025$$

Each of the layers is taken to be .025 in. thick and thus the dimensionless distances from the bottom of the first layer are given by

$$s_1 = 0.,\ s_2 = 0.25,\ s_3 = 0.5,\ s_4 = 0.75,\ s_5 = 1.0$$

Equation (26)

As mentioned previously, each layer of the symmetric angle ply configuration (elastic symmetry axes y are oriented at ($+\gamma$, $-\gamma$, $-\gamma$, $+\gamma$) is taken to be orthotropic with engineering elastic coefficients representing those for a boron/epoxy material system,

$$E_1 = 35\text{x}10^6 \text{ psi}, \qquad E_2 = 2.75\text{x}10^6 \text{ psi}$$

$$G_{12} = 0.75\text{x}10^6 \text{ psi}, \qquad \nu = 0.25$$

Equation (27)

Here direction 1 signifies the direction parallel to the fibers while 2 is the transverse direction. Angles chosen were = 0 , 15, 30, 45 and 60. Use of the results of Appendices A and B and the transformation equations (6) then yields the mechanical properties for the different symmetric angle ply configurations.

We next apply the following edge loads:

$$N = p$$

and take

$$\sigma = p/\lambda$$

$$H = (3/4)h$$

Shown in Figs. 3 - 5 is the variation of the dimensionless radial displacement with the actual distance along the axis for the different theories. The reference surface for the chosen configuration is given by

$$d/h = 1/2$$

As mentioned above, the theory associated with length scales a is a membrane type theory and it's radial displacement is a function of the dimension less pressure and the two integration constants determined from the edge conditions (12). As Fig.3 demonstrates the radial displacement of this theory is constant over the entire length of the shell. The variation of the magnitude of radial displacement due to the change of cross-ply angle is almost identical compared to the other theories except for the fact that the theory cannot describe the deformation pattern due to the boundary conditions while the other theories showing the radial deformations of the so-called edge effect zone. The theory associated with longitudinal length scale $(ah)^{1/2}$ and circumferential length scale a is similar to the axi-symmetric version of the theory of length scales $(ah)^{1/2}$ in the following aspects:

a) Expressions for strains and curvatures are identical as they were shown in (14) and (26). This is due to the fact that the theory of length scales $(ah)^{1/2}$ are much simplified by the axi-symmetric property while the other theory is closer in fashion to the axi-symmetric behavior by it's nature because of the larger circumferential length scale we used for the theory, i.e. a.

b) Although the expressions for the particular solutions are different as indicated in (24) and (33), the combined form of governing equations and the homogeneous solutions, as shown in (20) and (32), are identical in form. This is due to the same length scales being used in longitudinal direction, $(ah)^{1/2}$, for both theories and again, the axial symmetry. In obtaining the homogeneous solutions for both theories, it was assumed that the cylinder has such material properties and geometric dimensions so as to justify the decay type solutions (20) and (32). In order to have these decay type solutions we first must have that the value of term as shown in (22) must be real.

$$N_1N_3 - N_2^2 > 0$$

Secondly, the dimensionless shell length (must be sufficiently larger compared to the axial length scale used in the basic formulation of the theories, $(ah)^{1/2}$, so that interaction effects from the opposite edges may be neglected. The condition for satisfying this can be obtained by comparing the two decay terms in equation (20), exp $(-N_5\, x)$ and exp $(-N_5 5\, \zeta)$.

This leads to

$$\ell > \Gamma$$

where

$$\Gamma = \pi(ah)^{1/2}(N_1/N_3)^{1/4}$$

The restriction of the cylinder length **l** to be larger than **r** is important in the analysis of cylindrical shells due to the difference, of nature of the solution. For the cylinder shorter than r, the edge conditions have an effect on each other and the solution is no longer of the decay type. Edge conditions in this case govern the deformation pattern as well as the magnitude. A short cylinder under external pressure and closed at both ends deform axi-symmetrically and can be considered a typical example of a problem where the solution has a decay length shorter than r. It must be noted here that unlike for isotropic homogeneous shells, the decay length **r** depends not only the shell geometry, h and a, but also on the material properties of each laminate.

CONCLUSION

As stated previously, Figs. 4 and 5 show that the radial displacement of the shell at distances from the edge greater than r, from now on called the edge effect boundary layer, is nearly identical for both theories and close in magnitude to that of the solution which is obtained for length scales a. This is because, in the regions away from, the particular part of the solutions of governing equations dominate while the homogeneous solutions are more important within the boundary layer regions. Because the results shown in the figures are nearly identical for the problem considered, no numerical calculations of the uniformly valid solution given above is carried out.

It is also seen that wide variations in the magnitude of radial displacement take place with change in the cross-ply angle. The maximum displacement occurs at

γ =30 degree while the minimum displacement is at
γ = 60 degree. In each case, the displacements increase with increase in γ up to
γ = 30 degree and thereafter decrease. The attached Figures show that the edge effect is sharper for small angle G than for larger ones. Similarly, deeper penetration of the edge effect is shown for small angles γ while weak and smooth edge effects are the case for large cross-ply angles.

Shown in Figs. 6 and 7 are the dimensionless displacement of an isotropic material of elastic coefficient $30\text{x}10^6$ psi and in Figs shown are of single layer boron/epoxy composite we used for four layers case. Circumferential component of stress resultant is also shown in Figures

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