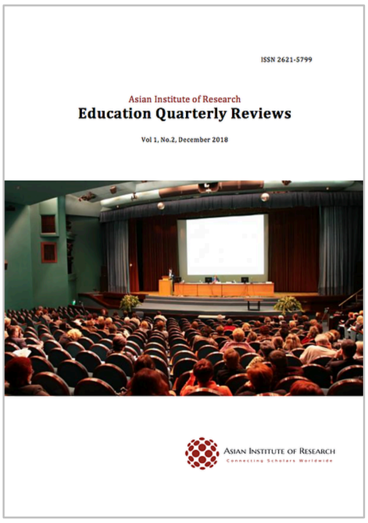

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Visual Teaching Strategies to Reduce the Learning Difficulties in Advanced Algebra and Analytic Geometry Course

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Abstract
In this article, we present the results of a classroom action study that was done in the course of advanced algebra and analytic geometry at Huaiyin Institute of Technology. Specifically, we identify and analyze the students' difficulties in learning the course of advanced algebra and analytic geometry, put forward three visual strategies to reduce the learning difficulties. We conduct a questionnaire after a final exam to determine how these visual teaching strategies help students to learn and how the visual teaching strategies affect their attitude towards advanced algebra and analytic geometry. The results are positive and help to answer the question about the value of visual teaching strategies, suggesting ways forward in teaching advanced algebra and analytic geometry.

Keywords: Advanced Algebra and Analytical Geometry, Visual Teaching, Learning Difficulty, Teaching Strategy

1. Introduction

With the continual development of advanced education reform in China, especially the college course reform, how to make the college students acquire more knowledge with less time and make them be intellectuals with stronger innovation ability, is a core subject that needs to be considered carefully in the process of college course reform. Traditionally, the most important and core curriculums for college students whose majors are mathematics in China are advanced algebra, analytic geometry, and mathematical analysis. But with the establishments of some new courses, the class hours of the previous core curriculums are less and less. And due to the fact that there are some close relationships between advanced algebra and analytic geometry, it is suitable and necessary to combine these two courses, advanced algebra, and analytic geometry, into one course. The first practitioner who teaches advanced algebra and analytic geometry as one course are Daoji Meng, a professor at Nankai University. The textbook (Meng, 1998) named after advanced algebra and analytic geometry was published by Science Press. Since then, more and more universities in China adopted this course teaching reform, and more textbooks come into the world.

The purpose of this paper is to identify and analyze the students' difficulties in learning the course of advanced algebra and analytic geometry, put forward three visual strategies to reduce learning difficulties. A questionnaire is conducted after a final exam to determine how the visual teaching strategies help students to learn advanced algebra and analytic geometry and how the visual teaching strategies affect their attitude towards advanced algebra and analytic geometry. The results are positive and help to answer the question about the value of visual teaching strategies, suggesting ways forward in teaching advanced algebra and analytic geometry.

2. The traditional teaching

In general, the curriculum of advanced algebra and analytic geometry is divided into two parts which are taught in the first and second semester respectively. Normally, the advanced algebra and analytic geometry course are taken simultaneously with the course of mathematical analysis. About 70 students who have taken an introductory math course with some logic and set theory, function and some proof techniques attend this advanced algebra and analytic geometry course yearly. The textbook used in the teaching of advanced algebra and analytic geometry is written by a group of teachers (2005) in the department of applied mathematics in Tongji University.

The teaching of the advanced algebra and analytic geometry course is fairly traditional in appearance. Its teaching process consists of lecturing, exercise including homework for practicing and reinforcing the learning, and examination including mid-semester and final examination for testing whether the students have learned some theories and procedures. As far as the lecturing is concerned, it presents some concepts, theories, and examples. In more detail, lecturing consists of definitions, properties and lemmas, theories and their proofs, examples, and commentaries on the exercises that have been done in the class. The course in the first semester lasts 14 weeks, and we have three 90 minute lectures a week. But in the second semester, the course lasts 15 weeks, and we have two lectures a week. One lecture takes 90, and the other takes 135 minutes. The polynomial theories of one indeterminate, analytic geometry, the systems of linear equations and matrix algebra are what the students expect, and they learn this course well. However, as far as our students are concerned, the advanced algebra and analytic geometry course are very difficult to learn as a whole. Taking the 70 students as an example, about 78.6% students have a lot of difficulty in learning this course, only 21.4% students think this course is easy to learn, and about 58.6% students are not interested in the course.

3. Learning difficulties

It is clear that poor and average students have a great deal of difficulty in learning this course for the first time. In order to enhance the teaching and learning of this course, it is natural for us to know something about what difficulties that our students have encountered in the process of teaching and learning advanced algebra and analytic geometry.

Table 1: *Level of the Learning difficulties in advanced algebra and analytic geometry*

level of learning the difficulty	extremely difficult	specially difficult	very difficult	fairly difficult
abstract concept	47	19	3	1
theoretical calculation	2	13	39	14
theorem's proof	18	34	17	0
lack of basic knowledge	3	3	11	51

The extreme difficulty which means the most difficult aspect is that there are many abstract concepts which are defined formally and axiomatized with many mathematical expressions (signs). As shown in Table 1, there are 47 students (of the total 70 students) who think so, and only 19 (respectively, 3 and 1) students think many abstract concepts is especially (respectively, very and fairly) difficult. It seems that many abstract mathematical concepts and definitions are the most important difficulty which our students have encountered.

On the one hand, many students have a lot of difficulty in understanding mathematical concepts and definitions in the course of advanced algebra and analytic geometry. It is known that almost all concepts and definitions in this course are defined formally and axiomatized with many mathematical expressions such as linear space and subspace, linearly dependence and independence, the direct sum of subspaces, isomorphism, dual space, linear transform, and inner product space, etc. Weak to average students tend to avoid the use of formal definitions. On the contrary, they try to use some context-dependent recipes, substitutes for actual definition, which come from the numerous but limited examples that they have been introduced to in the school mathematics.

On the other hand, since there are many mathematical relationships between these abstract concepts are not always clear, many students (especially, poor and average students) are usually confused and disoriented. For example, it is known that two matrices A and B are said to be equivalent if there are non-singular matrices P and Q such that $B=PAQ$, two matrices A and B are said to be similar if there are non-singular matrix S such that $B=SAS^{-1}$, where S^{-1} denotes the inverse of matrix S, and two matrices A and B are said to be congruent if there are non-singular matrix S such that $B=SAS^T$, where S^T denotes the transpose of matrix S. Those relationships between the above three concepts (equivalence, similarity, and congruence) are confusing for our students. Many students often failed to make these concepts' differences and connections clear.

Mathematical proof of a theorem is the second obstacle which is especially difficult. It is clear from Table 1 that nearly 100% of students think theorem's proof is at least very difficult to understand and learn, and 50% students think the mathematical proof is at least especially difficult. Recall that a proof of a theorem is a finite sequence of claims, each claim is derived logically from the previous claims, as well as theorems whose truth have been already established. The last claim in the sequence is the statement of the theorem, or the statement that clearly implies the theorem. It is well known that there are two different conceptions of mathematical proof. The first one is an informal proof, a rigorous natural-language expression that is intended to convince the audience of the truth of a theorem. The second one is a formal proof which is not written in a natural language, but instead uses a formal language consisting of certain strings of symbols from a fixed alphabet. In our teaching practice, almost all mathematical proofs of theorems are written in a formal language. The forms of theorems are multifarious, such as "If...Then...", "If...Then...Otherwise...", and "... if and only if ...", Etc. Methods of proof are also multifarious. Direct proof, proof by mathematical induction, proof by contradiction, proof by transposition which establishes the conclusion "if *p* then *q*" by proving the equivalent contrapositive statement "if not *q* then not *p*", proof by construction (or proof by example) which is the construction of a concrete example with a property to show that something having that property exists, and proof by exhaustion in which the conclusion is established by dividing it into a finite number of cases and proving each one separately, are applied widely in the proving processes of theorems and exercises. Most students tend to prove a theorem directly and are not familiar with other strategies of proving a theorem.

The third difficulty that is very difficult is that theoretical calculations and computations. It is known that theoretical calculations are usually tedious and cumbersome. It can be seen from Table 1 that nearly 60% of students think the tedious theoretical calculation is very difficult.

4. Visual teaching strategies

The term visualization is used in different meaning between mathematics educators. It is defined by Zazkis, Dubinsky, and Dautermann (1996), that is, as an act in which an individual establishes a strong connection between an internal construct and something to which access is gained through the sense. Such a connection can be made in two directions. An act of visualization may consist of any mental construction of an object or process that an individual associates with objects or events perceived by an external source. Alternatively, it may consist of the construction, on some external medium such as paper of objects or events. Consequently, the act of visualization is the translation from external to mental. As stated by Konyalioglu, Ipek and Isik (2005), visualization can be alternative method and powerful resource for students learning mathematics (especially, advanced algebra and analytic geometry), a resource that can upon the way to different ways of thinking about mathematics than the linguistic and logico-propositional thinking of traditional and the symbol manipulation of traditional algebra. Visual teaching method provides students to look at the course of advanced algebra and analytic geometry which was seen as an accumulation of abstract structures and concepts from a different perspective.

The first visual teaching strategy is to relate algebraic statements of these concepts to geometric statements. Graph, diagram, pictures and geometrical shape or models are tools for visualization of the abstract concepts and definitions in advanced algebra and analytic geometry. By means of these, a relation between physical or external world and the abstract concepts are established. Consequently, abstract concepts and definitions become concrete and clear for students to understand.

For instance, a geometric interpretation of the Gram-Schmidt orthogonalization process in R^3 not only helps students to understand the process but also prevents them from getting lost in the computations. Geometric interpretation of eigenvalues and eigenvectors of linear transformations in R^2 and R^3 has the same advantages. In a word, an integrated method that combines geometric interpretation and algebraic interpretation makes the students' learning and understanding better.

The second visual teaching strategy is to emphasize practicability. For example, Digital Image Processing (DIP) can be used widely in the teaching of advanced algebra and analytic geometry. It is known from the book written by Gonzales and Woods (2008) that an $m \times n$ digital image can be stored as an $m \times n$ matrix:

$$f(x, y) = \begin{matrix} f(0,0) & f(0,1) & \cdots & f(0,n) \\ f(1,0) & f(1,1) & \cdots & f(1,n) \\ \vdots & \vdots & & \vdots \\ f(m-1,0) & f(m-1,1) & \cdots & f(m-1,n-1) \end{matrix} ,$$

Where two-dimensional function $f(x, y)$ is called the intensity or grey level of the image at the point (x, y) and each element of the matrix is called a picture element or pixel. Consequently, all of the standard ways to manipulate images (contrast enhancement, lightening, darkening, histogram analysis, fading, warping, etc.) can be performed by applying mathematical operations to the matrix associated with the image. Since the results of such operations are inherently visual, students are guided by increasingly sophisticated visualizations, thus developing important problem-solving skills for the understanding of mathematical concepts and theories. Supported by modern DIP technology, the connection between the course of advanced algebra and analytic geometry needs to be exploited more efficiently. Students are intrigued by stretching, rotating or flipping images, and proceed naturally to the study of contrast enhancement, detecting images in data corrupted by noise, feature extraction, edge detection and techniques of data compression, and all these topics are fruitfully researched by using concepts from the course of higher algebra and analytic geometry. For instance, students can be introduced to a singular value decomposition method to compress data so that the original image could be reconstructed with much less data.

The third visual teaching strategy is to use computer software (e.g., Maple, Matlab, Mathematica, etc.). By using computer soft wares, we can free class time to teach students how to write proofs. Since coherent writing is not possible without coherent thinking, this is a useful teaching tool for students. It also leads to an appreciation of beauty in mathematics. Some students have been observed comparing different proofs and noticing the elegance of the shorter and simpler proof. Moreover, instead of simply giving examples of objects that satisfy a definition, or checking that they satisfy a certain list of properties, students are led by using computer software through a series of exercises to work with examples and explore their properties. For example, non-commutativity and other properties of the product of matrices become more real if students produce examples by themselves. Of course, computer software is not necessary to accomplish this purpose, but its use makes it possible to do this in class without spending too much time. Later on, students notice what the computer will not do for them: interpret the solutions of a linear system in geometric terms or in terms of the applied problems that gave rise to the system. This forces them to focus on understanding the process and the results instead of mechanically performing calculations.

5. Questionnaire and results

A questionnaire is conducted after a final exam to determine how visual teaching helps students to learn advanced algebra and analytic geometry and how the visual teaching affects their attitude towards advanced algebra and analytic geometry. The questionnaire is listed below with the results.

5.1 Questionnaire

Evaluate the following statements by choosing from (1) to (5), where:

(1) Strongly agree
(2) Agree
(3) Neutral
(4) Disagree
(5) Strongly disagree

Statement 1: *Visual teaching strategies made me think critically and more deeply about the concepts, theorems and their proofs in higher algebra and analytic geometry.*
Statement 2: *Visual teaching of higher algebra and analytic geometry improved my mathematical skills.*
Statement 3: *Visual teaching of higher algebra and analytic geometry changed my attitude towards mathematics.*

5.2 Results

Table 2. *Results of the questionnaire*

Statement	Strongly agree(%)	Agree (%)	Neutral (%)	Disagree(%)	Strongly disagree(%)
1	35	51.2	10	2.5	1.3
2	32.5	55	11.2	1.3	0
3	46.3	53.7	0	0	0

As seen from Table 2, at least 86% of students agreed that visual teaching strategies made them think critically and more deeply about the concepts, theorems and their proofs in advanced algebra and analytic geometry. 86.5% of students agreed that visual teaching of advanced algebra and analytic geometry improved their mathematical skills. To our surprise, all the students agreed that visual teaching of advanced algebra and analytic geometry changed their attitude towards mathematics positively.

6. Conclusions

The visual teaching strategies presented in this paper seek to reverse students' negative perceptions about advanced algebra and analytic geometry and to improve their course performance by developing their visual understanding of advanced algebra and analytic geometry. However, visual teaching is not meant to replace traditional topics. It provides a course supplement with a much different flavor to enhance the teaching of advanced algebra and analytic geometry.

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