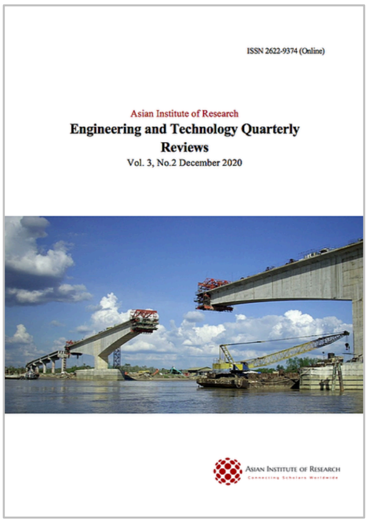

Engineering and Technology Quarterly Reviews

Sherin, Hemati. (2020), Eigenvalue of Adjacent Matrix of Zero Divisor Graphs on Rings. In: *Engineering and Technology Quarterly Reviews*, Vol.3, No.2, 63-66.

ISSN 2622-9374

The online version of this article can be found at:
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Published by:
The Asian Institute of Research

The Asian Institute of Research
Engineering and Technology Quarterly Reviews
Vol.3, No.2, 2020: 63-66
ISSN 2622-9374

Eigenvalue of Adjacent Matrix of Zero Divisor Graphs on Rings

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Abstract
Let R be a commutative ring with identity $1 \neq 0$ and T be the ring of all $n \times n$ upper triangular matrices over R. The zero-devisor graph of T denoted by $\Gamma(T_n(R))$. In this paper, I define the adjacent Matrix of $\Gamma(R)$ and $\Gamma(T_n(R))$. Then I describe the relation between the non-zero Eigenvalues of adjacent Matrix of these graph and edges. After I use these result to determination of Eigenvalue adjacent matrix of $\Gamma(T_2(R))$.

Keywords: Eigenvalue, Adjacency Matrix, Zero Devisors Graph of Commutative Ring

1. INTRODUCTION

A graph is described by its adjacency matrix and we can analyses different problem with count its eigenvalue and other properties. Adjacent Matrix of a graph were first defined by Harary (Harary 1962). Jorgensen characterized the parameter sets for all directed strongly regular graphs with adjacent matrix of rank 3 or 4 (Jørgensen 2005).

Let R be a commutative ring with identity $1 \neq 0$and $ZD(R)$denote the set of zero- devisors of R. The zero-devisor graph of R, denoted $\Gamma(R)$, is the undirected graph. There is an edge in $\Gamma(R)$between the vertices r and s if and only if $rs = 0$. Using the language of graph theory, the set of vertices of $\Gamma(R)$ is $V(\Gamma(R)) = ZD(R)$ and the set of edges of $\Gamma(R)$is $E(\Gamma(R)) = \{(r,s) \setminus rs = 0\}$.

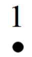
Zero-divisor graphs were first defined for commutative rings by Beck (Beck 1988) when the coloring of graphs was studied, and later redefined by Anderson and Livingston (Anderson and Naseer 1993). Redmond (Redmond 2002) introduced the concept of the zero -divisor graph for a non-commutative ring R. Bapat describe the relation between the distinct Eigenvalues of adjacent Matrix a graph and its diameter (Bapat 2010) . In this paper I introduced the relation between Eigenvalue and numbers of edges graph $\Gamma(R)$. Further extend result to graph $\Gamma(T_2(R))$.

2. Eigenvalue of the adjacent matrix of zero divisor graph on commutative ring

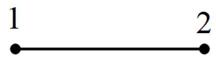
I start with some examples from reference (Anderson and Livingston 1999) motivate next result.

Example 2.1. (a) Assume ring R is $R = \frac{Z_2[x]}{\langle x^2 \rangle}$ or non-isomorphic rings R . Then $ZD(R) = \{\bar{0}, \bar{x}\}$ and

$\Gamma(R)$ has one vertex, adjacent matrix of $\Gamma(R)$ is $[0]_{1\times 1}$.

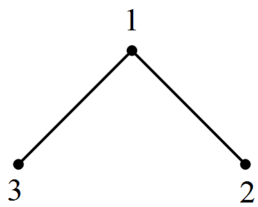


(b) Assume ring R is $R = \frac{Z_3[x]}{\langle x^2 \rangle}$ or non-isomorphic rings R. then $ZD(R) = \{\bar{0}, \bar{x}, 2\bar{x}\}$ and $\Gamma(R)$, adjacent matrix of $\Gamma(R)$ is the follow.



$$\begin{array}{c} \\ 1 \\ 2 \end{array}\begin{array}{c} \begin{array}{cc} 1 & 2 \end{array} \\ \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}_{2\times 2} \end{array}$$

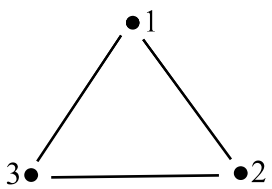
(c) Assume ring R is $R = \frac{Z_2[x]}{\langle x^3 \rangle}$ or non-isomorphic rings R. then $ZD(R) = \{\bar{0}, \bar{x}, \overline{x^2}, \overline{x + x^2}\}$ and $\Gamma(R)$, adjacent matrix of $\Gamma(R)$ is the follow.



$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}_{3\times 3} \end{array}$$

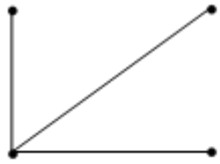
(c) Assume ring R is $R = \frac{Z_2[x, y]}{\langle x^2, xy, y^2 \rangle}$, $R = \frac{F_4[x]}{\langle x^2 \rangle}$ or non-isomorphic rings R. then

$ZD(R) = \{\bar{0}, \bar{x}, \bar{y}, \overline{x + y}\}$ or $ZD(R) = \{\bar{0}, \bar{x}, \overline{a_1 x}, \overline{a_2 x} / a_1, a_2 \in F_4\}$ and $\Gamma(R)$, adjacent matrix of $\Gamma(R)$ is the follow.

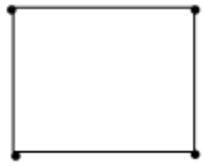


$$\begin{array}{c} \\ 1 \\ 2 \\ 3 \end{array}\begin{array}{c} \begin{array}{ccc} 1 & 2 & 3 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}_{3\times 3} \end{array}$$

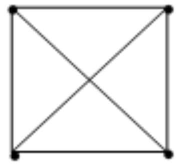
By part (a), (b), (c) all connected graphs with less than four vertices may be realized as $\Gamma(R)$. Of the eleven graphs with four vertices, only six are connected. Of these six, only the following three graphs may be realized as $\Gamma(R)$.



$\Gamma(\mathbb{Z}_3 \times \mathbb{Z}_3)$



$\Gamma(\mathbb{Z}_2 \times \mathbb{F}_4)$



$\Gamma(\mathbb{Z}_{25}) \subsetneq \Gamma(\frac{\mathbb{Z}_5[x]}{\langle x^2 \rangle})$

We can easily proof that the $\Gamma(R)$ can be a triangle or a square. But, $\Gamma(R)$ cannot be an n- gon for any $n \geq 5$.

Lemma 2.2. Let R be a commutative ring with identity and $\Gamma(R)$ is zero divisor graph of ring R with number of vertices $n \leq 3$ and non-cyclic, then its non-zero Eigenvalue of adjacent matrix is equal square root numbers of edges.

Proof. If graph has one vertex, then result is clear. If graph has two vertices, then

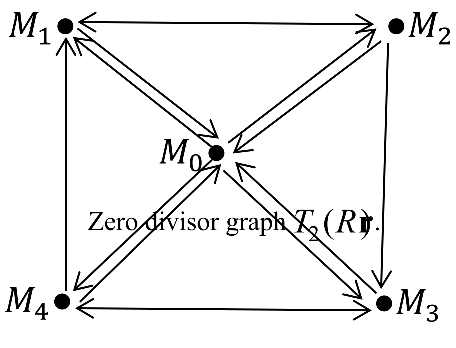
$$|A-\lambda I| = 0 \Rightarrow \left| \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right| = \begin{vmatrix} -\lambda & 1 \\ 1 & -\lambda \end{vmatrix} = \lambda^2 - 1 = 0 \Rightarrow \lambda = \pm\sqrt{1}.$$

If graph has three vertices, then

$$|A-\lambda I| = 0 \Rightarrow \left| \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right| = 0 \Rightarrow \begin{vmatrix} -\lambda & 1 & 1 \\ 1 & -\lambda & 0 \\ 1 & 0 & -\lambda \end{vmatrix} = 0 \Rightarrow \lambda = \pm\sqrt{2}.$$ □

3. Eigenvalue of the adjacent matrix of zero divisor graph on triangular matrix ring

At first by use reference (Li and Tucci 2013) ,we study zero divisor graph of $T_2(R)$.in general, every element of $T_2(R)$ will be denoted $\begin{bmatrix} x_1 & x_2 \\ 0 & x_3 \end{bmatrix}$ where each $x_j \in R$.



Zero divisor graph $T_2(R)$.

And adjacent matrix of $T_2(R)$ is follow that:

$$A(T_2(R)) = \begin{array}{c} \\ M_0 \\ M_1 \\ M_2 \\ M_3 \\ M_4 \end{array} \begin{array}{c} \begin{array}{ccccc} M_0 & M_1 & M_2 & M_3 & M_4 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}_{5\times 5} \end{array}$$

Theorem3.1. Let $\Gamma(T_2(R))$be zero divisor graph of triangular matrices 2×2. Then Eigenvalue of the adjacent matrix this graph are 1with multiplicity 1 and -2, 0 with multiplicity 2.

Proof. By the properties of determinant, rank this matrix is not 5 or 4. Now we study determinant matrix 3×3.

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \end{vmatrix} = (-1)^{1+2}(1)\begin{vmatrix} 1 & 1 \\ 1 & 0 \end{vmatrix} = -[(0-1)] = 1.$$

So rank $A(T_2(R))$ =3. Therefore this matrix has 3 non- Eigenvalues.it follows that

$$\Rightarrow \lambda^4 + 3\lambda^3 - 4\lambda = 0 \Rightarrow \lambda_1 = \lambda_2 = 0, \lambda_3 = \lambda_4 = -2 \quad \wedge \quad \lambda_5 = 1. \qquad \square$$

4. Conclusion

In conclusion, I define adjacent matrix of $\Gamma(R)$ and study relation between Eigenvalue and numbers of edges this graph. I will show that $\Gamma(R)$with numbers of vertices $n \le 3$and non-cyclic, its non-zero Eigenvalue of adjacent matrix is equal square root numbers of edges. Further extend result to determination of Eigenvalue adjacent matrix of $\Gamma(T_2(R))$.

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