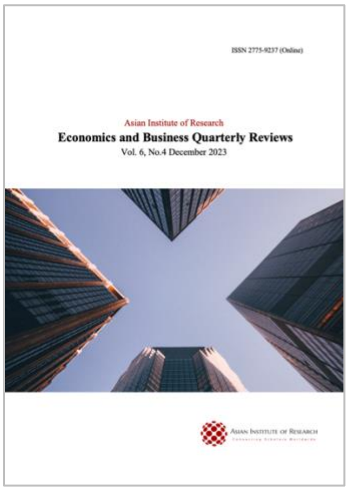

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Simple Estimations of the Natural Rate of Interest in Japan with the Band-Pass Filters

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Abstract
Estimation of natural rate of interest is required since it is unobservable. One of the practical ways of estimating the natural interest rate is to extract the trend component from estimated short-term real interest rate. We utilize the Christiano-Fitzgerald filter and the Baxter-King filter for the extraction process. If the central bank intends to raise the policy interest rate as a policy implementation when it acknowledges an inflationary pressure (or excess demand), and if this policy action is smoothly transmitted into adjustment processes of prices in economic activities, statistically observed short-term real interest rate should behave closely around the evolutionary path of the natural interest rate. In this context, the extracted trend component from short-term real interest rate by means of a statistical filter would be regarded as the estimated natural rate of interest. We utilize the Christiano-Fitzgerald filter and the Baxter-King filter for our extraction process. Concerning the estimation of expected inflation rate as the factor to construct the short-term real interest rate, the Carson-Parkin method is adopted. Our empirical results show a recent persistent decline of natural rate of interest in Japan.

Keywords: Natural Rate of Interest, Inflation Expectations, Christiano-Fitzgerald Filter, Baxter-King Filter

JEL Classification Code: C53; E31; E32, E37

1. Introduction

To conduct an empirical research on the natural interest rate is an inevitable work for economists since it is one of the significant determinants or references for monetary policy. In general, the natural rate of interest is regarded as the short-term real interest rate consistent with the output equaling its potential level or the one reflecting stable prices. Since it cannot be observed directly, estimation of the natural rate of interest is required. Because of these properties, many elaborate previous studies on theoretical and empirical analysis have been published.

The issue whether the natural interest rate is substantially declining (and will continue to decline) has attracted attention after the noticeable remarks by Summers (2013). Rachel and Smith (2015), Sajedi and Thwaites (2016), and Eggertsson *et al.* (2017) as well as Summers (2014) are the examples of research that are related to this issue or to the secular stagnation hypothesis.

Laubach and Williams (2003) is one of the most famous research of this kind. It regards the IS curve, the Phillips curve, and the natural rate of interest equations as the keys to describe the structural relationships of economy, and uses the Kalman filter to estimate some unobservable variables. Pescatori and Turunen (2016), Hakkio and Smith (2017), Holston *et al.* (2017), and Lewis and Vazquez-Grande (2017), Fries *et al.* (2018) are the applications of Laubach and Williams (2003). On the other hand, Lubik and Matthes (2015) applies the time-varying parameter vector autoregressions method.

The so-called DSGE (dynamic stochastic general equilibrium) model that clarifies structural interpretations of the system is also a commonly used instrument for analyzing the natural rate of interest. Edge *et al.* (2008), Justiniano and Primiceri (2010), Barsky *et al.* (2014), Curdia *et al.* (2015), Del Negro *et al.* (2015, 2017), Goldby *et al.* (2015), Hristov (2016), Gerali and Neri (2017), Okazaki and Sudo (2018), and Iiboshi *et al.* (2022) are the studies based on DSGE methodology.

One of the informative methods to estimate the natural interest rate is to extract the trend component of short-term real interest rate by utilizing a statistical filter. Namely, if the central bank is inclined to raise the policy interest rate when it acknowledges an inflationary pressure (or excess demand) and if such a policy implementation is transmitted smoothly into adjustment processes of prices in economic activities, short-term real interest rate should behave closely around the evolutionary path of the natural rate of interest. In this sense, extracted trend of short-term real interest rate through a statistical filter would be regarded as the estimated value of the natural rate of interest. We utilize the Christiano-Fitzgerald filter and the Baxter-King filter for the extraction process. On the other hand, we should have the estimation of expected inflation rate in order to acquire short-term real interest rate by following the Fisher equation. In this paper, the Carlson and Parkin (1975) method is adopted to estimate the expected inflation rate.

The rest of this paper is organized as follows. Section 2 summarizes the Carlson-Parkin method, and the estimation of consumers' inflation expectation. Section 3 sets out the empirical investigations of the natural rate of interest by utilizing the Christiano-Fitzgerald (2003) filter and the Baxter-King (1999) filter. Lastly, Section 4 is devoted to the concluding remarks.

2. Inflation Expectations and its Estimations

2.1. Survey Data for the Estimation

The main purpose of this research is to estimate the natural rate of interest based on short-term real interest rate. In this regard, we should have the estimation of expected inflation rate in the course of acquiring the short-term real interest rate in accordance with the Fisher equation. In short, the Fisher equation explains that the real interest rate equals the nominal interest rate minus the expected inflation rate.

Consideration of the inflation formation process from the aspect of general public's expectation with survey data would lead us towards widely applicable research without any specific economic models. We can find two types of survey data on inflation expectations: qualitative and quantitative. In the qualitative surveys, respondents answer in a qualitative manner. The data given by this kind of survey is presented in a qualitative statistical form that indicates whether the majority of the polled respondents expect that future price level will rise, remain constant, or decline. In other words, the survey of this type explores the general tendency of the expectation on the future price level or inflation. On the other hand, respondents answer to the question in a quantitative manner in the case of a quantitative survey. The problem in the case of quantitative survey is that it is hard to obtain an exact point forecast of the public's inflation expectation because the survey of this kind may have some flaws. For instance, this kind of direct measure is apt to cause some measurement and sampling errors. Thus, it is worth relying on the special method of quantifying qualitative data.

2.2. The Carlson-Parkin Method for Quantifying Qualitative Survey Data

We need a method of quantifying qualitative data series to examine public's inflation expectation expressed as the answer to the question in the social survey. Our difficulty is the fact that the way of manipulation of the data given by a qualitative survey with respect to prices can generate some problems. For example, the respondents answer

Table 1: Example of the Survey Result of "Price Expectations a Year Ahead"

(Unit:%)

	Go down			Stay the same	Go up			Don't know
	greater than or equal to -5%	less than -5% to greater than or equal to -2%	less than -2%	about 0%	less than 2%	greater than or equal to 2% to less than 5%	greater than or equal to 5%	
2012 Jul	1.7	2.6	4.4	19.2	19.1	30.0	16.2	6.9
Aug	1.3	2.1	5.6	21.5	22.1	26.6	14.0	6.8
Sep	1.1	1.9	5.0	18.5	23.2	29.2	14.9	6.2
Oct	0.8	2.5	4.4	17.0	25.4	31.0	13.6	5.3
Nov	0.7	1.9	5.6	20.4	25.0	27.7	13.3	5.3
Dec	0.7	2.4	6.3	20.8	24.1	26.2	13.6	5.8

Source: http://www.esri.cao.go.jp/en/stat/shouhi/shiken_summary_e.html

only their expectation of price level (or inflation) as "rise", "fall" or "remain unchanged" for some periods ahead in the survey. Namely, because they are qualitative data, we cannot get a statistical value. Therefore, several techniques have been proposed to deal with this difficulty. The Carlson and Parkin (1975) method is the most useful probability approach for quantifying qualitative data and often applied to estimate rate of expected inflation.

The Carlson-Parkin method presumes that the qualitative answer in the social survey follows an individual probability distribution that is statistically independent from the one of other respondents and normally distributed with finite mean and variance. Furthermore, this method assumes that, at time t, respondents form an inflation expectation for time $t+1$. The joint probability distribution $f(\mathrm{x}_{t+1}|\Omega_t)$ is derived by the aggregation of the individual subjective probability distributions where x_{t+1} is the percentage change in price level for the period from t to $t+1$ and Ω_t describes the information set at time t.This distribution is considered to have finite first- and second-order moments, and we can express the inflation expectation for the period $t+1$ as $E[\mathrm{x}_{t+1}|\Omega_t] = \pi^e_{t+1}$. In addition, Carlson-Parkin methodology postulates that there exists a threshold, δ_t (>0), for the interval $(-\delta_t, \delta_t)$ around 0 in order that the answer "no change" will lies within this interval.

We utilize the Carlson-Parkin method to estimate the expected inflation in Japan. However, it is not exactly the same as the original procedure but the modified one. The modified version for our estimation is based on Henzel and Wollmershäuser (2006), Oral (2013), and Scheufele (2011).

2.3. Consumer Confidence Survey

The "consumer confidence survey" (in Japan), conducted by the Economic and Social Research Institute of the Cabinet Office,[1] asks the respondents to report consumer perception and expectation about the economy.[2] It is one of the most useful data sources for empirical study with the Carlson-Parkin-type approach in Japan. Concretely, monthly data series of the survey are available from April 2004 onward. In particular, the qualitative data indicated in the section "price expectations a year ahead" would be applied to our estimation of inflation expectations in that the respondents give their expectations of future price level as "go down," "stay the same," "go up," or "don't know." The example of the survey result is displayed in Table 1.

[1] See "http://www.esri.cao.go.jp/en/stat/shouhi/shouhi_kaisetsu-e.html" for details.
[2] The things that we should pay attention to the "consumer confidence survey" are as follows. (a) The survey of "price expectations a year ahead" is conducted across three categories: "all households," "all households except one-person households," and "one-person households." (b) From May 2004 to February 2007, telephone surveys were conducted in months other than June, September, December, and March, and direct-visit and self-completion questionnaires are conducted in June, September, December, and March. (c) Since April 2013, the survey has been conducted by mail. In addition, the number of sample households has been increased. Therefore, we are obliged to use discontinuous survey data in our sample period.

Figure 1: Estimated Expected Inflation Rate with the Carlson-Parkin Method based on Japan's "consumer confidence survey" (%)

2.4 Estimation of Consumers' Inflation Expectations

Monthly data of "consumer price index" (excluding fresh food, whole Japan, total), seasonally non-adjusted,[3] change from the previous year, spanning the period from April 2004 to October 2023[4] is utilized as the rate of inflation (π_t) for our estimation. On the other hand, the qualitative data obtained from the "consumer confidence survey" for "all households" is used for our constitution of expected inflation rate (π_t^e) with the Carlson-Parkin method. The three ratios respectively included in the "go down" and "go up" (see Table 1) are combined to make the totals of "go down" and "go up" for our estimation, while the answers "don't know" are eliminated from our data set. Thus, the ratios for each item are reorganized in accordance with our revised format.

Figure 1 reports the estimated expected inflation rates by the Carlson-Parkin method with $\hat{\delta} = 0.101573$.[5] The estimated expected inflation rate is fluctuated through our sample period from the lowest: $-0.010873(\%)$ for December 2009 to the highest: $1.461125(\%)$ for February 2023.

3. Estimating the Natural Rate of Interest with Statistical Filters

3.1. Basics of the Christiano-Fitzgerald Filter

We can decompose the series into a trend and a cyclical component to understand behavior of time series data. The so-called "band-pass filter" is used to separate the cyclical component of a time series data by specifying a certain range of its duration. To put it another way, the band-pass filter is one of the statistical linear filters with a two-sided weighted moving average of the data series where cycles in a specified "band" is consisted by the lower and the upper limits. The series classified into the "band" are extracted and "passed" through the "band-pass filter" whereas the remaining cycles are filtered out.

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[3] The reason of adopting seasonally non-adjusted series (instead of seasonally adjusted series) of consumer price index (change from the previous year) is as follows. We also have to use the data given by the consumer confidence survey which are not seasonally adjusted. In order to have a consistency of the format between the two kinds of data series, seasonally non-adjusted series of consumer price index is applied.

[4] The data on "consumer price index" were retrieved from the "Portal Site" of Official Statistics of Japan operated by the Ministry of Internal Affairs and Communications, Statistics Bureau, Director-General for Policy Planning (Statistical Standards) & Statistical Research and Training Institute (in English): "http://www.e-stat.go.jp/SG1/estat/eStatTopPortalE.do."

[5] Several problems regarding sign of δ have been discussed in previous studies.

There are some kinds of band-pass filter, for example, the Christiano-Fitzgerald filter,[6] the Baxter-King filter,[7] and so on. The important point is that all band-pass filters that practically used are only approximations of the "ideal" filter. The "ideal" band-pass filter is designed as a both-side or two-sided infinite-order moving average process:

$$a(L) = \sum_{k=-\infty}^{\infty} a_k L^k. \tag{1}$$

The cycle, in short, the filtered series, is found by passing the raw series through the filter,

$$\varepsilon_t = a(L) y_t, \tag{2}$$

where y_t and ε_t are the unfiltered and the filtered series, respectively.

In the frequency domain, the transfer function can be recovered by the rule,

$$T(\omega) = \left| a(e^{-i\omega}) \right|^2, \tag{3}$$

where ω is the frequency. The corresponding spectrum of the cycle is

$$s_\varepsilon(\omega) = T(\omega) s_y(\omega), \quad \omega \in [0, \pi], \tag{4}$$

where s_ε is the spectrum of the filtered series, and s_y is the one of the original series. $T(\omega)$ is the weight of the frequency responses as they pass to s_ε from s_y.

The "ideal" transfer function is described by

$$T(\omega) = \begin{cases} 1 & \omega_l \leq |\omega| \leq \omega_u \\ 0 & othewise, \end{cases} \tag{5}$$

where ω_l and ω_u are the lower and the upper limits on the frequencies of interests. The "ideal" transfer function passes through a band of frequencies into the spectrum of the cycle unaltered. However, the "ideal" band-pass filter based on equation (1) needs an infinite amount of information to construct. Therefore, the "ideal" transfer function has to take an approximate structure in a practical sense.

For this reason, in the case of the Christiano-Fitzgerald filter, the moving average process is truncated as

$$\hat{T}^{p,f}(L) = \left| \sum_{j=-f}^{p} \hat{a}_j^{p,f} L^j \right|^2, \tag{6}$$

where $f = T - t$ and $p = t - 1$. The $\hat{a}_j^{p,f}$ are the weights and given if we solve the following minimization problem:

$$\min_{\hat{T}_{j, j=-f,\dots,p}} \int_{-\pi}^{\pi} \left| T(e^{i\omega}) - \hat{T}^{p,f}(e^{i\omega}) \right|^2 s_y(\omega) d\omega, \qquad \textit{subject to } \hat{T}^{p,f}(1) = 0. \tag{7}$$

This specification is close to the one of the Baxter-King filter with some exceptions. Actually, the objective function in equation (7) is identical to the one of the Baxter-King filter except for the $s_y(\omega)$ weighting $\left| T(e^{i\omega}) - \hat{T}^{p,f}(e^{i\omega}) \right|^2$ term. If y_t is white noise with a flat spectrum, the objective function in equation (7) is compatible with the one in the Baxter-King filter. On the other hand, the objective function in equation (7) does not follow the symmetry constraint applied in the Baxter-King filter. The constraint $\hat{T}^{p,f}(1) = 0$ expresses that the filter returns stationary results, while the relaxation of the symmetry constraint follows a phase shift. The filtered time series by Christiano-Fitzgerald filter is to be trimmed, not on both-side, but only on the left side. Namely, the Christiano-Fitzgerald filter is taken advantage of the real-time applications where the current estimate of the cycle is desirable. In contrast, the Baxter-King filter trims both sides of the filtered series, and it not always optimal for real-time forecasting.

The solution of the minimization problem expressed by the equation (7) is described as

$$c_t = a_0 y_t + a_1 y_{t+1} + \cdots + a_{T-1-t} y_{T-1} + \tilde{a}_{T-1} y_T + a_1 y_{t-1} + \cdots + a_{t-2} y_2 + \tilde{a}_{T-1} y_1$$
$$\text{for } t = 3, 4, \dots, T-2, \tag{8}$$

where $a_j = \frac{sin(jc) - sin(jb)}{\pi j}$ for $j \geq 1$, $a_0 = \frac{c-b}{\pi}$, $b = \frac{2\pi}{p_h}$, $c = \frac{2\pi}{p_l}$, and $\hat{a}_k = -\frac{1}{2} a_0 \sum_{j=1}^{k-1} a_j$.

[6] See Christiano and Fitzgerald (2003) for details.
[7] See Baxter and King (1999) for details.

3.2. Basics of the Baxter-King Filter

The so-called Baxter and King (1999) filter is one kind of band pass filter as a modification of the Hodrick-Prescott (1997) filter. It is a method of extracting cycle component through signaling out the repeated component of a time series setting the width for oscillations of periodic component.

Baxter-King filter is a bandpass filter of finite order K which is optimal if it is an approximate bandpass filter with trend-reducing properties and symmetric weights ensuring that there is no phase shift in the filter output. The impact of the filter on an input series y_t in time domain is obtained by the finite moving average:

$$y_t = B(L)x_t, \tag{9}$$

$$B(L) = \sum_{j=-\infty}^{\infty} B_j L^j, \tag{10}$$

$$L^n x_t = x_{t-n} \tag{11}$$

where L is the lag operator. In a frequency domain, the filter is characterized by its Fourier transform, $\alpha(\omega)$, and computation of the weights needs the cut-off frequency which is set by the user describing permissible non-cyclical (or non-seasonal) oscillation of the smoothed series. Or, in order to find the weights B_j, we should solve the minimization problem as

$$\min_{a_j} Q = \int_{-\pi}^{\pi} |\beta(\omega) - \alpha(\omega)|^2 s_y(\omega) d\omega, \qquad \textit{subject to } \alpha(0) = 0; \tag{12}$$

where $|\beta(\omega)|$ is the gain of the "ideal" filter with cut-off frequencies ω_1 and ω_2. The gain of a filter describes the change in the amplitude of the input components if it is transformed by the filter. The gain of the "ideal" bandpass filter, $|\beta(\omega)|$, takes the value 1 in the $[\omega_1, \omega_2]$ frequency interval, or 0 outside this interval. For the two kinds of ω, we define them as

$$\omega_1 = \frac{2\pi}{\omega_u};\ \omega_2 = \frac{2\pi}{\omega_l} \tag{13}$$

where ω_u, ω_l are the upper and the lower limits of the cut-off frequency of our interests. The constraint ensures that the resulting filter has trend reducing properties. If we remove the component with the frequency $\omega = 0$ from the series, then the filter weights must sum to zero[8].

Solving the minimization problem leads to the following results:

$$B_j = b_j + \theta,\ j = 0, \pm 1, \dots, \pm K, \tag{14}$$

$$b_0 = \frac{\omega_2 - \omega_1}{\pi}, \tag{15}$$

$$b_j = \frac{sin(\omega_2 j) - sin(\omega_1 j)}{\pi j} \quad \text{if } j = \pm 1, \pm 2, \dots, \tag{16}$$

$$\theta = \frac{-\sum_{j=-K}^{K} b_j}{2K+1}. \tag{17}$$

The filter is symmetric (namely, $B_j = B_{-j}$). Thus, it does not impose a phase shift on the output. Considering the experience with the business cycle in the united states, Baxter-King filter proposes the set of parameters: $K = 12$, $\omega_1 = 2\pi\frac{1}{32}$, and $\omega_2 = 2\pi\frac{1}{6}$ or $2\pi\frac{1}{2}$ for quarterly data, while it suggests $K = 3$, $\omega_1 = 2\pi\frac{1}{8}$, and $\omega_2 = \pi$ for annual data. However, these values depend on the length of the observation period and on the frequency band. The power transfer function (ptf) of the Baxter-King filter is its squared gain, and allows to evaluate the filter impact on the spectrum of the input series.

The Baxter-King filter removes the cycle component S from the time series Y by passing on the weighted moving average with specified weight. A cycle (or seasonal) component of the source series is given by the formula:

$$S_t = Y_t B_0 + \sum_{j=1}^{K} Y_{t-j} B_j + \sum_{j=1}^{K} Y_{t+j} B_j, \tag{18}$$

where B_j means the weight value in accordance with value of the source series Y at the distance j from the current element. The result of smoothing by the filter is the source series with removed cycle component. A generalized Baxter-King filter can be applied to non-stationary time series. In this case, "non-stationarity" is described by the matrix of weights that depends on the number of observations in generalized model. Thus, computation of cycle component is implemented by the formula:

$$S_t = Y_t B_{0,t} + \sum_{j=1}^{K} Y_{t-j} B_{j,t} + \sum_{j=1}^{K} Y_{t+j} B_{j,t}, \tag{19}$$

[8] See Baxter and King (1999) for details.

Table 2: Augmented Dickey-Fuller Test

without drift and trend				
variable	level / first difference	lag Length	test statistic	p-value
estimated real short-term interest rate	level	0	-0.306587	0.5744
estimated real short-term interest rate	first difference	0	-16.41822	0.0000
with drift				
variable	level / first difference	lag length	test statistic	p-value
estimated real short-term interest rate	level	0	-1.108676	0.7128
estimated real short-term interest rate	first difference	0	-16.43397	0.0000
with drift and trend				
variable	level / first difference	lag length	test Statistic	p-value
estimated real short-term interest rate	level	0	-2.607941	0.2772
estimated real short-term interest rate	first difference	0	-16.43004	0.0000

Notes: The p value (one-sided) is based on MacKinnon (1996). Optimal lag length is determined by the Schwarz information criterion with the maximum length = 12.

where $B_{j,t}$ is the weight value that correspond to value of the source series Y at the distance j from the element Y_t. The Baxter-King filter proposed the recommended values for lead, lag, upper and lower limits of bandwidth based on frequency of the source series.

3.3. Estimation of the Natural Rate of Interest

Estimation of natural rate of interest is required since it is unobservable. One of the practical ways of estimating the natural interest rate is to extract the trend component from short-term real interest rate. We utilize the Christiano-Fitzgerald filter and the Baxter-King filter for the extraction process.

If the central bank intends to raise the policy interest rate as a policy implementation when it acknowledges an inflationary pressure (or excess demand), and this policy action is smoothly transmitted into adjustment processes of prices in economic activities, statistically observed short-term real interest rate should behave closely around the evolutionary path of the natural interest rate. In this context, the extracted trend component from short-term real interest rate by means of a statistical filter would be regarded as the estimated natural rate of interest. Concerning the estimation of expected inflation rate as the factor to construct the short-term real interest rate, the Carson-Parkin method described in the previous section is adopted. Concretely, we assume the following relation by considering the Fisher equation, and estimated the short-term real interest rate.

Estimated short-term real interest rate
= short-term nominal interest rate − expected inflation rate
= observed uncollateralized overnight call rate[9] − estimated expected inflation rate with the Carson-Parkin method (explained in Section 2.2)

We implement the trend extraction based on this estimated short-term real interest rate. The Christiano-Fitzgerald (2003) filter is utilized for our first extraction. We use the full sample and asymmetric lag type Christiano-Fitzgerald filter (although symmetric fixed forward and backward lags type also can be applied). Before this process, the unit root test to find the time series characteristics of the estimated short-term real interest rate should be conducted. Concretely, the augmented Dickey-Fuller (ADF) tests are implemented to check the order of integration of the variable. According to the test results shown in Table 2, the test statistic is not significant at level but it is significant at first difference. These test results imply that the estimated short-term real interest rate is integrated of order 1 (written as $I(1)$), or it follows a unit root process. In line with this result, the Christiano-Fitzgerald filter (full sample and asymmetric type) is applied with the assumption that the variable (short-term real

[9] The data on "uncollateralized overnight call rate" were retrieved from "BOJ Time-Series Data Search" in the website of The Bank of Japan (in English) "https://www.stat-search.boj.or.jp/index_en.html".

Figure 2: Estimated Natural Interest Rate with the Christiano-Fitzgerald Filter (full sample and asymmetric lag type) (%)

interest rate) follows $I(1)$ unit root process. The fixed length filters consume same number of lead and lag terms for all weighted moving average, and lose the same number of observations from the beginning and the end of the original sample. In contrast, the asymmetric length filters can conduct an estimation from the beginning to the end of the sample without losing any observations. The asymmetric-type filter is operated with different weights on the lead and lag terms. They are time-varying depending on the data and changing for each observation. The duration or periodicities range (P_L, P_U) to pass through should be set when we utilize the band-pass filters including the Christiano-Fitzgerald filter and the Baxter-King filter. One way to settle (P_L, P_U) is (18, 90) for monthly observation by following Burns and Mitchell (1946) since they assume business cycle endures somewhere from 1.5 to 8 years. However, in our study, we assume (P_L, P_U) is (31, 90) based on "The Reference Dates of Business Cycle"[10] decided by the Committee for Business Cycle Indicators (organized by the Economic and Social Research Institute, Cabinet Office, Government of Japan) since we find the shortest duration of business cycle in Japan is 31 months (the 8th cycle) and the longest one is 90 months (the 16th cycle) as of November, 2023.

Next, the Baxter-King (1999) filter is used for our second extraction process. It is originally a symmetric fixed lag type filter, and we set symmetric fixed 12 forward and backward lags in this study. The fixed length symmetric filter including the Baxter-King filter needs a fixed lead/lag length for weighted moving average. This type of filters is time-invariant because the weights for moving average rely on the specified frequency band, not on the data. With regard to the (P_L, P_U), we assume (31, 90) based on "The Reference Dates of Business Cycle" as is the case with the estimation by utilizing the Christiano-Fitzgerald filter described above.

Figure 2 shows the estimated natural interest rate by applying the Christiano-Fitzgerald filter (with full sample) as the non-cycle series. The estimated rate goes back and forth between positive and negative values around zero from April 2004 to March 2010. Subsequently, it consistently takes negative value from October 2010 to October 2023 although it fluctuates and shows unstable developments.

Figure 3 describes the estimated natural rate of interest by utilizing the Baxter-King filter. We lose 12 observations from both the beginning and end of the sample since we set 12 fixed leads and lags for our filter. By considering the result of estimation, we find out that the estimated natural rate of interest shows quite unstable movement around zero percent from April 2005 to December 2010, and it consistently takes negative value from January 2011 to October 2022, the end of observation.

[10] See "https://www.esri.cao.go.jp/en/stat/di/rdates.html" and "https://www.esri.cao.go.jp/jp/stat/di/hiduke.html" for details.

Figure 3: Estimated Natural Interest Rate with the Baxter-King Filter (fixed length symmetric type) (%)

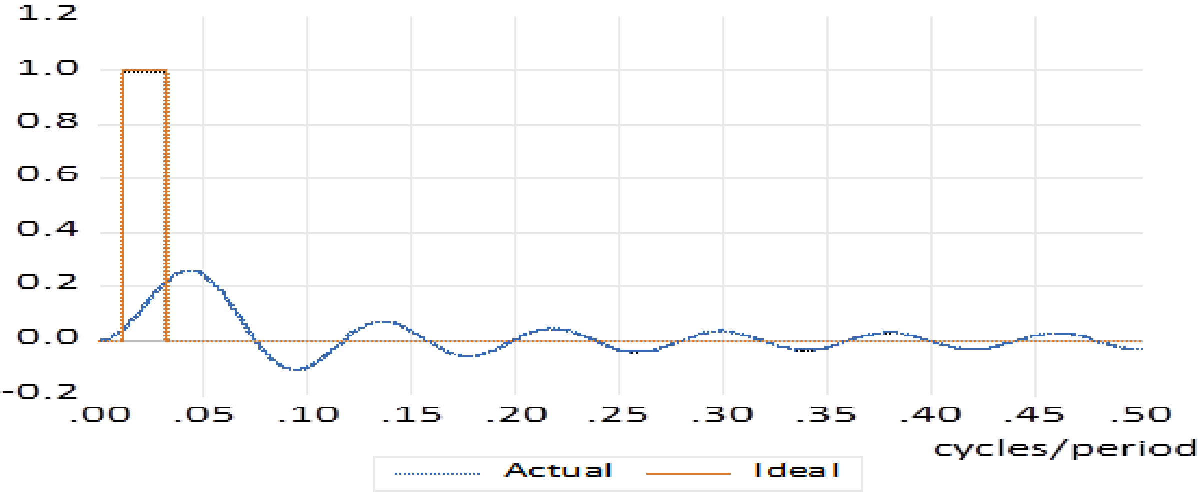

Figure 4: Frequency Response Function of Estimated Natural Rate of Interest by the Baxter-King Filter

Figure 4 displays the frequency response function of the estimated natural interest rate derived by the Baxter-King Filter. The horizontal axis is in the interval between 0 to 0.5 in units of cycles per duration, and the frequency response function given by the "ideal" band-pass filter including the Baxter-King filter for periodicities (P_L, P_U) should be in the range $\left(\frac{1}{P_U}, \frac{1}{P_L}\right)$. For a frequency ω, we have $|\alpha(\omega)|^2$ that expresses how the moving average raises and lowers the variance of the filtered series in comparison with the one given by the original series. In this context, $\alpha(\omega)$, the frequency response function, describes the response of the filtered series to the original series at frequency ω. The derived response depicted in Figure 4 does not seem to be close to the one for the "ideal" filter. It may indicate a potential problem of our estimation.

If these results of estimation are reliable information, the Japanese central bank is faced with the great difficulty since it is hard to deal with such a situation by conducting a monetary policy based on the usual framework.

4. Concluding Remarks

Our empirical results show a recent persistent decline of natural rate of interest in Japan. If the natural rate of interest continues to fall. usual framework of monetary policy cannot be pursued. In a situation like this, monetary policy has no choice but to guide real interest rate to negative level by raising expected inflation rate or by decreasing nominal interest rate. The difficulty is whether the central bank can induce real interest rate to be negative when the natural interest rate is negative with the non-negative constraint of nominal rate. In this context, detecting the cause of decline of the natural rate of interest with the consideration of its future direction should be an integral element for deliberating effective monetary policy in Japan.

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