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Applying Capability Indices with Moving Average and Range Control Charts to Evaluate a Product in a Local Aluminum Plant

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Abstract

The study of production processes capability is one of the most important statistical process control techniques (SPC). This study allows answering an important question; is the manufacturing system capable of producing a product with fewer defects and according to the wishes and requirements of consumer and customer? The main objective of this study is the assessment of a process capability and indices for the production process of aluminum sections regarding to weight, hardness, coating, tensile strength and heat treatment in a local aluminum plant, these aluminum sections and parts are widely used in industry, building, rails, and many other applications. The study was carried out in one of the local aluminum factory for the production of specific engineering sections, some capability indices were applied to the most affective characteristics on the quality of these produced sections; as weight, hardness, coating, tensile strength, heat treatment, these indices were applied after confirming the statistical control of the production process using the quality control charts, specifically moving average and range control charts.

Keywords: Capability Indices, Defects, Heat Treatment, Control Charts

1. Introduction

A process has been well-defined as a series of interdependent methods, operations, or steps consuming resources and changing the inputs to outputs. Every step or operation is added to the following for achieving an aim or a favorite result. In each process, there is a specific quantity of variation that cannot be removed, but it can be monitored, measured, decreased and controlled. When one looks at a simple instant of preparing a cup of coffee, one can specify the steps, inputs, apparatus and the process output.

The necessary steps (as inputs) require switching on the coffee producer, measure and add the water as well as coffee, and then the output being a cup of coffee. The variation can take place in the quantity of water or coffee presented in this process and the coffee producer performance. Each cup of coffee is not precisely the similar but in the majority states, when measurements are precise and sensibly consistent, it flavors similar. Via using the controls of process, taking the precise measurements and utilizing reliable, highly maintained apparatuses, the process variation can possess less influence upon the output quality.

In order to make the process able to produce an acceptable product on a reliable basis, Process Capability (Cp) can be used. Cp is a statistical measurement of the ability of a process for producing parts within identified specification on a consistent basis. For determining how the process is working, one can compute the followings: Cp, Process Capability Index (Cpk), Preliminary Process Capability (Pp), and Preliminary Process Capability Index (Ppk), based upon the process condition and the procedure of obtaining the value of standard deviation (or sigma). The computations of (Cp) and (Cpk) utilize the sample deviation or the deviation of mean within the rational sub-groups. The computations of (Pp) and (Ppk) employ the standard deviation depending upon the investigated studied data (the whole population). The indices of (Cp) and (Cpk) are utilized for evaluating the current, set processes in the statistical control. The indices (Pp) and (Ppk) are employed for evaluating a fresh process or one, which isn't in the statistical control.

The (Cp) and (Cpk) assess the process output compared to the limits of specification obtained via the value of target and the range of tolerance. (Cp) says when the process is able to make parts within the specifications, and (Cpk) expresses at the point when the process is focused between the specification limits. If engineers design components, they ought to regard the capability of machine or the process chosen to manufacture the component. For illustration, if one uses an actual world instant, imagine one drives a vehicle over a bridge. The width vehicle width is equal to the data spread or range. The guardrails on every side of bridge are the limits of specification. One ought to keep the vehicle on bridge for reaching the other side. The value of (Cp) is equal to the distance that the vehicle keeps away from the guardrails, and the (Cpk) denotes how well one is driving down the mid of bridge. Clearly, when the data spread is narrower (the width of car is smaller), the more distance exists between guardrails and vehicle and the more probable one stays on bridge.

The (Cp) index is an important sign of the capability of process. The value of (Cp) is computed utilizing the limits of specification and the process standard deviation. Major of companies need that the process ($Cp \geq 1.33$). The process center's (Cpk) index moves a step more via testing how near a process is achieving to the limits of specification regarding the usual variation of process. The bigger value of (Cpk), the nearer the data mean being to the value of target. The (Cpk) is computed utilizing the limits of specification, standard deviation (or sigma), and the magnitude of mean. The value of (Cpk) has to be within (1) and (3). When this value is < 1 , the process should be improved.

The (Cp) and (Cpk) indices are merely as good as the data utilized. The accurate capability of process investigations is relied on (3) rudimentary assumptions concerning the data:

There're no distinctive variation causes in process and it's in a statistical control state. Some distinctive causes ought to be revealed and determined.

The data fits a normal distribution, displaying a bell-shaped curve and can be computed to (\pm three sigma). There're states if the data doesn't fit a normal distribution.

Data of sample is a population's representative. Data has to be arbitrarily gathered from a big run of manufacture. Numerous companies need at minimum (25) to rather (50) measurements of sample be gathered.

2. Process Capability

Process capability is described as the process inherent variability in the nonappearance of every undesirable causes; the least variability, of which the process being fit with the variability owing only to the regular causes.

Distinctively, the processes pursues the distribution of normal probability, it that's correct, the process measurements high percentage drops between ($\pm 3\sigma$) of the mean or center of process.

That's, a nearly (0.27%) of the measurements would normally drop outside the limits of ($\pm 3\sigma$), and the stability of them (nearly 99.73%) would be inside the limits of ($\pm 3\sigma$).

Because the limits of process range from (-3σ) to ($+3\sigma$), the whole spread quantities to around (6σ) entire variation. When one compares the spread of process with the spread of specification, one distinctively has one of the three following states [3]:

State I:

A truly competent process, the spread of process is fine inside the spread of specification $6\sigma < (USL - LSL)$, Fig.1.stateI

Where:

USL: Upper specification limit

LSL: Lower specification limit

σ : Process standard deviation

State II:

A hardly capable process, the spread of process is just around matching (6σ), Fig.1.stateII

Where:

$$6\sigma = (USL - LSL)$$

If a spread of process being just around equal to the spread of specification, this process is capable of satisfying the specifications, but hardly so. This proposes that when the mean of processes shifts just slightly to the left or to the right, an important output quantity will surpass one of the limits of specification. This process should be observed carefully for detecting the movements from mean. The control charts are brilliant way for doing this.

State III:

The spread of process is more than the specification range, Fig.1.stateIII

Where:

$$6\sigma > (USL - LSL)$$

I

f the process of process is larger than the range of specification, this process isn't capable of satisfying the specifications irrespective of where the mean or center of process is situated. That's really a sorry state, often this takes place, and the responsible people aren't even conscious of it.

The over process modification is one result, leading to even more variability; the substitutes comprise [1]:

- Varying the process to a high dependable technique or investigating the process cautiously in a try to decrease the variability of process.
- Living with the present process and sorting (100%) of products.
- Re-centering the process for minimizing the whole losses outside the limits of specification.
- Ending the process and getting out of such business.

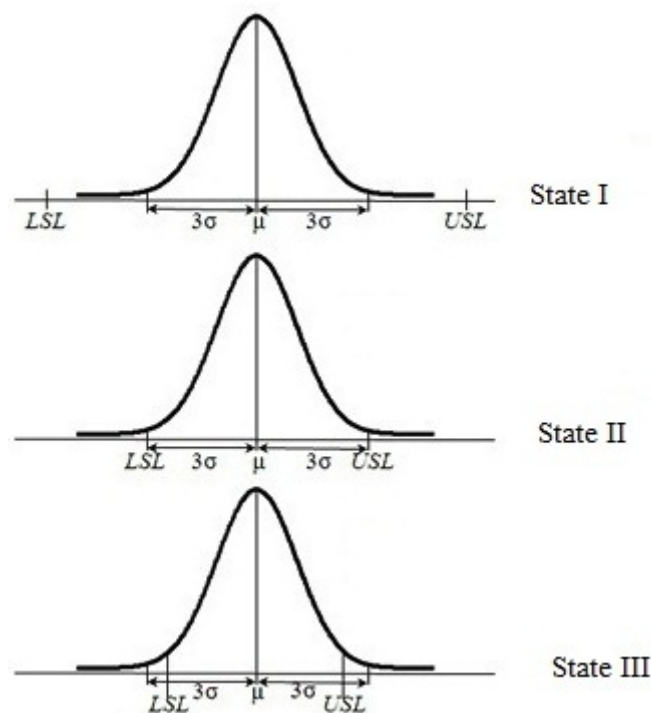


Figure1. States of process capability

3. Capability Indices

These are simple measures for rapidly describing the relation between process variability and the specification limits spread.

Similar to numerous simple measures, like the stages (A), (B), (C), (D), and (F) in university, the capability indices don't totally define what is taking place with the process.

They're beneficial if the assumptions for utilizing them are satisfied for comparing the capabilities of processes.

Capability indices (C_p , C_{pu} , C_{pl} , C_{pk} , and C_{pm})

The term of the simple capability index (C_p) is the ratio of the spread of specification to the spread of process; the last is characterized via (6) standard deviations or (6σ).

$$C_p = \frac{USL - LSL}{6\sigma}$$

Where:

$100(1/C_p)$ is construed as the specifications' width percentage utilized via process.

The earlier (C_p) equation assumes that the process possesses both the upper and the lower limits of specification. Nevertheless, numerous applied cases can offer merely a single limit of specification. In such situation, the one-sided (C_p) is described via [6]:

$$C_{pu} = \frac{USL - \mu}{3\sigma} \quad (\text{Upper specification only})$$

$$C_{pl} = \frac{\mu - LSL}{3\sigma} \quad (\text{Lower specification only})$$

One can notice that the (C_p) measures the centered process capability. But, the whole processes aren't essentially be usually centered at the nominal dimension, that means, the processes may also go off-center, then the real capability of processes that are non-centered will be lower than that directed via (C_p). In a situation if the process is going off-center, the process capability being measured via the pursuing ratio (Process Capability Index C_{pk}):

$$C_{pk} = \min [C_{pu}, C_{pl}]$$

$$C_{pk} = \min \left[\frac{USL - \mu}{3\sigma}, \frac{\mu - LSL}{3\sigma} \right]$$

- 1) If ($C_{pk} = C_p$), at that point this process is centered at the specifications midpoint.
- 2) If ($C_{pk} < C_p$), at that point this process is going off-center.
- 3) If ($C_{pk} = 0$), the mean of process is precisely equivalent to one of specification limits.
- 4) If ($C_{pk} < 0$), then the process mean locates outside the specification limits.
- 5) If ($C_{pk} < -1$), then the whole process locates outside the specification limits.

Sometimes, one has two production processes with different means and standard deviations, but they have the same value of specification limits and C_{pk} , as shown in Table1.

Table 1: Two production processes different means and standard deviations

Process	A	B
Mean	50.0 cN	57.5 cN
Standard deviation	5.0 cN	2.5 cN
Specification limits	35cN, 65cN	35cN, 65cN
C_p	1	2
C_{pk}	1	1

The way for addressing such difficulty is to employ the ratio of process capability, which is a better centering indicator. One such modified being [6]:

$$C_{pm} = \frac{USL - LSL}{6\tau}$$

Where:

T is the square root of the anticipated squared deviation from the target (T).

$$T = \frac{1}{2} (USL + LSL)$$

Then:

$$\tau^2 = \sigma^2 + (\mu - T)^2$$

$$C_{pm} = \frac{USL - LSL}{6\sqrt{\sigma^2 + (\mu - T)^2}}$$

$$C_{pm} = (USL - LSL) / 6\sigma \sqrt{1 + \left[\frac{\mu - T}{\sigma}\right]^2}$$

$$C_{pm} = C_p / \sqrt{1 + [(\mu - T) / \sigma]^2}$$

4. Moving Averages Control Charts

The (X) and (R) charts track the processes' performance that possesses lengthy runs of production or reiterated services, but occasionally there may be inadequate no. of sample measurements for creating the conventional (X) and (R) charts, for instant, just a single sample may be taken from a process. Instead of plotting every individual reading, it may be very suitable to implement the moving average and the moving range charts for combining (n) no. of the individual values for creating an average. If taking a fresh individual reading, the oldest value that forms the preceding average is rejected; the fresh reading being combined with the residual values from the preceding average for forming a fresh average.

That's too familiar in an uninterrupted process of chemical industry, where merely a single reading is likely at a time. Via the combination of the individual values made over time, the moving averages smoothen out the variations of short term and offer the trends in data, for such cause, moving average charts being often utilized for the seasonal outputs. The control limits of the moving average charts are the same of the traditional (X) and (R) charts, as follow:

$$\bar{\bar{X}} = \frac{\sum \bar{X}}{g}$$

$$\bar{R} = \frac{\sum R}{g}$$

$$UCL_X = \bar{\bar{X}} + A_2 \bar{R}$$

$$LCL_X = \bar{\bar{X}} - A_2 \bar{R}$$

$$UCL_R = D_4 \bar{R}$$

$$LCL_R = D_3 \bar{R}$$

Where:

g: Number of \bar{X}_i readings

5. Total Percentage outside the Specifications

For the purpose of determining the percentage of defects outside the upper and lower limit of standard, the value of (Z) must be determined for them as follows:

$$Z_{USL} = (USL - \bar{\bar{X}}) / \sigma$$

$$Z_{LSL} = (\bar{\bar{X}} - LSL) / \sigma$$

After determining the value of Z_{USL} and Z_{LSL} , the (Z-score) table is used to determine the percentage of defects outside the upper and lower limits of the standard.

6. Sections Produced in the Company

Arab Aluminum Industry Co. Ltd. (ARAL) was established in 1976, it was founded to achieve a fully integrated aluminum extrusion industry.

The ARAL features of the extensive fabricating facilities include:

- 1- A die-making plant and die-correction shop
- 2- A billet casting plant
- 3- Four hydraulic presses ranging from (640) to (2000) tons, with billet diameters ranging from (4) to (8) inches.
- 4- Two complete anodizing lines equipped with 72000 and 42000 amps rectifiers.
- 5- Two horizontal electrostatic paint lines.
- 6- A poly urethane thermal "Fill Lines".

The production facilities and manufacturing processes are shown in Table 2, and the different types of profiles that produced in this company are depicted in Fig. 2.

Table 2: Production facilities and manufacturing processes

Stages	Process	Process Description	Quality Inspection
1	Logs Manufacturing	Melting pure aluminum and placing it in a Cryolite bath.	
2	Logs Heating	Heating Logs in a three-heat zone furnace.	
3	Cutting Logs into Billets	Fully automated shear for the required length.	
4	Extrusion of Billets	Pressing billets through a heated die to obtain the required profile.	
5	Measuring Dimensions and Surface Finish.	Making graphite mark over the surface and using micrometer to ensure if it is accepted. If it is not, the die will be sent back to the die correction workshop to correct the error.	Check the dimensions and surface finishing
6	Weighing the Profile	The profile is weighed to ensure if it is accepted. If it is not, the die will be sent back to the die correction workshop to correct the error.	Check the weight
7	Aging for Accepted Profiles	Accepted profiles will be heat treated in the aging furnace for 8-10 hours at 190°C.	
8	Chemical Treatment	After aging, the profile will subject to a chemical treatment in another department	Check the oxide thickness
9	Rejected Profiles Recycling	Rejected profiles will be recycled by means of melting and casting again to take the logs shape.	

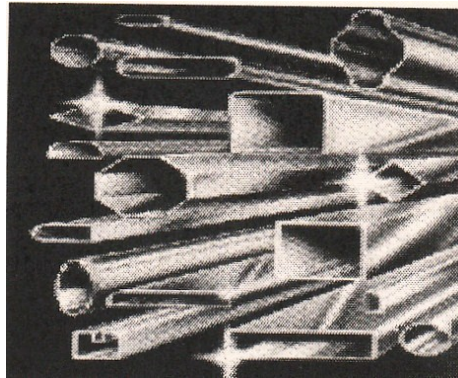


Figure 2: Sections produced in a company

In this study, one of these sections was selected to determine the process capability and quality indices on one of the properties specified in the specification (Weight property), see Fig. 3 and Table 3.

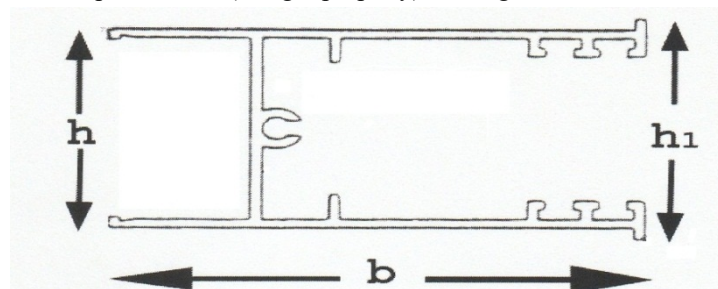


Figure 3: The selected section

Table 3: Specifications of the selected section dimensions

Property Specification	Weight (g/m)	b (mm)	h (mm)	h ₁ (mm)
USL	878	83	37.7	42
LSL	764	70	30	28

Moving average and moving range charts are selected for this purpose, thirty consecutive samples were drawn from the section in Fig.3 at specified intervals, and their dimensions and weights for one meter were recorded, for sample size=1, and g = 3, as shown in the Table 4.

Table 4: Subgroups weight and dimensions

Subgroups	Weight (g/m)	Dimensions (mm)			Subgroups	Weight (g/m)	Dimensions (mm)		
		b	h	h ₁			b	h	h ₁
1	790	73.2	34.7	37.5	16	849	79.8	34.2	36.8
2	787	70.1	33.9	38.9	17	790	70.5	35.5	41.6
3	820	78.3	35.4	39.1	18	813	84.2	36.4	39.1
4	843	79.5	32.7	37.8	19	834	78.0	37.1	40.3
5	85.1	80.2	36.4	39.7	20	773	81.7	32.3	35.5
6	796	72.3	33.5	38.6	21	851	86.2	37.3	41.8
7	868	84.1	35.1	40.2	22	880	82.5	37.1	41.3
8	805	71.6	35.8	37.4	23	805	74.1	36.0	38.7
9	813	77.0	35.7	41.0	24	817	69.3	32.9	38.8
10	781	69.3	33.2	36.7	25	795	72.5	35.2	41.2
11	850	78.7	34.1	43.2	26	821	77.4	34.5	35.3
12	810	76.5	33.7	39.8	27	498	75.1	38.0	39.2
13	832	70.4	34.9	37.4	28	817	73.2	33.9	38.8
14	843	77.2	34.7	37.8	29	848	81.3	35.0	40.7
15	821	80.1	35.8	41.0	30	833	76.7	36.7	39.8

The condition of determining the process capability of the production and its quality parameters is that the production process must be under statistical control, to achieve this requirement, quality control charts should be used to ensure or to reach a statistical control of the production process. Calculations for moving average and range control charts are illustrated in Table 5, and then the control limits are calculated as below:

Table 5: Calculated values for the moving average and range control charts

Subgroups	Weight (g/m)	3P Sum (W)	3P AVG (W)	3P Range (W)
1	790			
2	787			
3	820	2397	799	33
4	843	2450	816.67	56
5	851	2514	838	31
6	796	2490	830	55
7	868	2515	838.33	72
8	805	2469	823	72
9	813	2486	828.67	63
10	781	2399	799.67	32
11	850	2444	814.67	69
12	810	2441	813.67	69
13	832	2492	830.67	40
14	843	2485	828.33	33
15	821	2496	832	22
16	849	2513	837.67	28
17	790	2460	820	59
18	813	2452	817.33	59
19	834	2437	812.33	44
20	773	2420	806.67	61
21	851	2458	819.33	78
22	850	2474	824.67	78
23	805	2506	835.33	56

24	817	2472	824	45
25	795	2417	805.67	22
26	821	2433	811	26
27	798	2414	804.67	26
28	817	2436	812	23
29	848	2463	821	50
30	833	2498	832.67	31
		Sum	22977	1333

$$\begin{aligned} \bar{\bar{X}} &= 22977/28 & \bar{R} &= 1333 / 28 \\ \bar{\bar{X}} &= 820.6 \text{ g/m} & \bar{R} &= 47.6 \text{ g/m} \\ UCL_x &= \bar{\bar{X}} + A_2 \bar{R} & UCL_R &= D_4 \bar{R} \\ UCL_x &= 820.6 + (1.023 \times 47.6) & UCL_R &= 2.575 \times 47.6 \\ UCL_x &= 869.3 \text{ g/m} & UCL_R &= 122.57 \text{ g/m} \\ LCL_x &= \bar{\bar{X}} - A_2 \bar{R} & LCL_R &= D_3 \bar{R} \\ LCL_x &= 820.6 - (1.023 \times 47.6) & LCL_R &= 0 \times 47.6 \\ LCL_x &= 771.9 \text{ g/m} & LCL_R &= 0 \end{aligned}$$

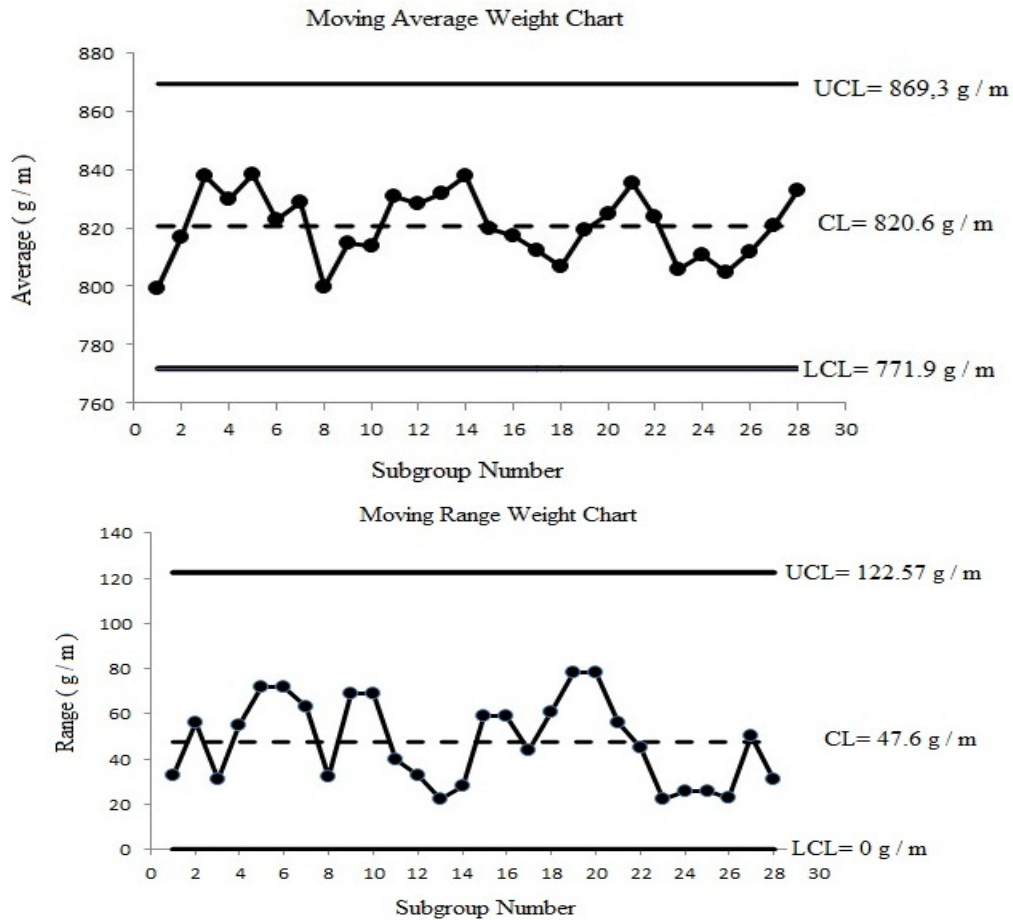


Figure4. Moving average and moving range control charts for profile ABS115 weight

From moving average and moving range control charts for profile ABS115 weight in Fig.4, it is clear that the production process is statistically controlled and there is no indication of the existence of assignable causes, thus one can determine the process capability and indices.

7. Process capability and indices for the weight of profile ABS115

The specification limits are $USL = 878 \text{ g/m}$ and $LSL = 764 \text{ g/m}$.

From this specification, one can say that the nominal mean value of process (T) is:

$$T = \frac{USL+LSL}{2}$$

$$T = (878+764) / 2$$

$$T = 821 \text{ g/m}$$

Now, the estimation of process standard deviation (σ) can be calculated:

$$\sigma = \frac{\bar{R}}{d_2}$$

$d_2 =$ Constant for the variable control charts depending on a sample size.

$$\sigma = \frac{47.6}{1.693}$$

$$\sigma = 28.11 \text{ g/m}$$

$$6\sigma = 6 \times 28.11$$

$$6\sigma = 168.66 \text{ g/m}$$

$$C_p = \frac{USL-LSL}{6\sigma}$$

$$C_p = \frac{878-764}{168.66}$$

$$C_p = 0.67$$

The specifications' width percentage utilized via the process = $100(1/C_p)$

The specifications' width percentage utilized via the process = $100(1/0.670)$

The specifications' width percentage utilized via the process = 149.25%

$$C_{pk} = \min \left[\frac{USL-\mu}{3\sigma}, \frac{\mu-LSL}{3\sigma} \right]$$

$$C_{pk} = \min [0.68, 0.66]$$

$$C_{pk} = 0.66$$

$$C_{pm} = C_p / \sqrt{1 + [(\mu-T)/\sigma]^2}$$

$$C_{pm} = 0.67 / \sqrt{1 + [(820.6-821)/28.11]^2}$$

$$C_{pm} = 0.67$$

8. Defective Percentage

$$Z_{USL} = \frac{878-820.6}{28.11}$$

$$Z_{USL} = 2.04$$

From (Z score) table, the number corresponding to 2.04 is 0.9773, for this:

The percentage of defective outside upper control limit = $1 - 0.9773$

The percentage of defective outside upper control limit = $0.0227 = 2.27\%$

$$Z_{LSL} = \frac{820.6-764}{28.11}$$

$$Z_{LSL} = 2.01$$

From (Z score) table, the number corresponding to 2.01 is 0.9778, for this:

The percentage of defective outside lower control limit = $1 - 0.9778$

The percentage of defective outside lower control limit = $0.0222 = 2.22\%$

Total defective percentage = $2.27+2.22$

Total defective percentage = 4.49%

9. Conclusion

It was observed that the production process was statistically controlled for the weight of the chosen section, but when measuring the process capability, the result was 0.67; this means that the production process occupies

149% of the standard specified for the weight of the section, for this, not all statistically controlled processes can produce products without defects, but the statistical control means that the production process is free of non-assignable causes and it works under assignable causes only.

It has been shown that the capacity of the capability of process (C_p) is larger than the process capability index (C_{pk}) value by about 0.01; this means the process is running off center, as mentioned before.

The production process goal is $T = 821\text{ gm/m}$, and the average production process deviates from it by 0.4 gm/m , as its value $\bar{X} = 820.6\text{ gm/m}$, and this is confirmed by $C_{pk} < C_p$.

It is possible to reduce or eliminate this slight deviation in the process average \bar{X} by shifting its value by 0.4 g/m towards the upper specification limit, but it will cause an increase in the defects percentage outside the upper limit of specification and reduce the defects percentage outside the lower limit of specification.

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